Algebra I: Quantities and Units

1. Kyle measured the distance from his house to the library. He discovered that his measurement had a margin of error of 0.05 mile. Which of the following could be the measured distance from his house to the library?

- **A.** 30.54 miles
- **B.** 30.5 miles
- **C.** 30 miles
- **D.** 30.543 miles

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2. Elton is cooking. He measures vegetable oil using a measuring cup that has increments of 1 ounce. Which of the following could be the volume of the vegetable oil with the correct number of significant digits?

- **A.** 7.569 ounces
- **B.** 7 ounces
- **C.** 7.5 ounces
- **D.** 7.57 ounces

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3. The density, *d*, of an object is given by the following formula, where *m* represents the mass of the object and *V* represents the volume of the object.

$$d = \frac{m}{V}$$

Heather was asked to find the density of a brick given the mass in kilograms and the dimensions of the brick in meters. She wrote her answer, 2,400 kilograms per square meter, on the chalkboard. Without even performing the calculations first, Jonathan knew right away that her answer was incorrect. How could he tell there was an error?

- **A.** The units should be kilograms.
- **B.** The units should be kilograms per cubic meter.
- **C.** The units should be cubic meters.
- **D.** The units should be kilograms per meter.

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4. A communications company that provides cable, Internet, and phone services, wants to measure its overall customer satisfaction of its services. Which measure provides the best information to the company?

- **A.** complaints per service
- **B.** complaints per customer per service
- **C.** complaints per year
- **D.** complaints per year per customer

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5. Kelly's recipe calls for 12 ounces of chicken. Her scale only measures in grams. Which of the following shows the number of grams of chicken, with the correct significant digits, that Kelly needs to include in her recipe in order to have 12 ounces? (1 ounce = 28.3495 grams)

- **A.** 339.6
- **B.** 341
- **C.** 340.194
- **D.** 340

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6. Each day, Karen makes 38 loaves of bread at the bakery where she works. Each loaf of bread requires 5 cups of flour. One pound of flour is equivalent to about 4 cups of flour.

About how many pounds of flour does Karen need in order to make the loaves of bread?

- **A.** 760
- **B.** 38
- **C.** 190
- **D.** 47.5

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7. In January, Melissa spent 25 minutes on her cell phone each day. If her monthly bill for the 31 days was $108.50, how much money does she pay per minute?

- **A.** $0.14
- **B.** $4.34
- **C.** $0.25
- **D.** $3.50

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8. A website displayed the following formula for the volume of a cone.

$$V = \frac{1}{3}\pi r h$$

James had never seen this formula before, but he knew there must be a typographical error in this formula. How could he tell there was an error?

- **A.** The calculated volume would be in square units instead of cubic units.
- **B.** The calculated volume would be in linear units instead of cubic units.
- **C.** The calculated volume would be in square units instead of linear units.
- **D.** The calculated volume would be in cubic units instead of square units.

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9. The luminosity of a star can be calculated by using the following equation.

$$L = 4\pi r^2 \sigma t^4$$

In the luminosity equation, L represents the luminosity in watts, r represents the radius of the star in meters, σ represents a constant equal to 5.67×10^{-8} $Wm^{-2}K^{-4}$, and t represents the surface temperature in Kelvins.

What is the approximate radius of a star with a luminosity equal to 2.532×10^{21} W and a surface temperature of 3,000 K? (Use $\pi = 3.14$)

- **A.** 4.5927×10^6 m
- **B.** 5.76843×10^7 m
- **C.** 6.62526×10^6 m
- **D.** 4.38941×10^{13} m

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10. Kinetic energy is the energy of a moving object. The kinetic energy of an object can be found by using the following formula, where m represents the mass of the object in kilograms and v represents the velocity of the object in meters per second.

$$\mathrm{E_k} = \frac{1}{2}mv^2$$

What is the kinetic energy of a 50 kg mass traveling at 22 meters per second?

- **A.** $12{,}100\ \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$
- **B.** $572\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$
- **C.** $550\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$
- **D.** $24{,}200\ \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$

Answers: Quantities & Units

1. B
2. C
3. B
4. C
5. D
6. D
7. A
8. A
9. C
10. A

Explanations

1. The margin of error is one-half the value of the last significant place.

Since Kyle had a margin of error of 0.05, the last significant digit in his measurement will be in the tenths place, as shown below.

$$\frac{1}{2} \times \frac{1}{10} = 0.05$$

Therefore, **30.5 miles** could be the measured distance from his house to the library.

2. Significant digits include all digits that are known for certain plus the last digit, which contains some uncertainty.

Since Elton's measuring cup has 1-ounce increments, he can know for certain the digits up to the ones place, plus the last digit in the tenths place, which will be an estimate.

Therefore, the measurement of the volume of vegetable oil will have two significant digits, with the last digit in the tenths place.

Thus, **7.5 ounces** could be the volume of the vegetable oil.

3. The volume of the brick is equal to the product of the length, width, and height of the brick.

$$\begin{aligned} d &= \frac{m \text{ kilograms}}{(l \text{ meters})(w \text{ meters})(h \text{ meters})} \\ &= \frac{m \text{ kilograms}}{(lwh) \text{ cubic meters}} \\ &= \left(\frac{m}{lwh}\right) \frac{\text{kilograms}}{\text{cubic meter}} \end{aligned}$$

Therefore, Jonathan could tell there was an error because **the units should be kilograms per cubic meter**.

4. To determine overall customer satisfaction, the best measure is **complaints per year**.

The other measures provide more detailed information that is not necessary for determining overall customer satisfaction.

5. The rules for significant digits are:

1. Non-zero digits are always significant.
2. Zeroes to the left of another digit are NOT significant.
3. Zeroes between non-zero digits are always significant.
4. Zeroes at the end of a number and to the right of a decimal point are always significant.
5. Zeroes at the end of a number, but to the left of a decimal point are NOT significant.

When multiplying or dividing with significant digits, the answer will have the same number of significant digits as the number with the LEAST significant digits.

$$12 \text{ ounces} \times 28.3495 \, \frac{\text{grams}}{\text{ounce}} = 340.194 \text{ grams}$$

The number of grams, 28.3495, in an ounce has six significant digits. The number of ounces of chicken, 12, has two significant digits.

Therefore, 340.194 rounded to two significant digits is **340** grams.

6. First, calculate the total number of cups of flour needed to make the 38 loaves.

$$\frac{1 \text{ loaf}}{5 \text{ cups}} = \frac{38 \text{ loaves}}{x}$$

$$x = \frac{190 \text{ (loaves} \cdot \text{cups)}}{1 \text{ loaf}}$$

$$x = 190 \text{ cups}$$

Next, find the weight of the flour.

$$\frac{1 \text{ pound}}{4 \text{ cups}} = \frac{y}{190 \text{ cups}}$$

$$\frac{190 \text{ (pound} \cdot \text{cups)}}{4 \text{ cups}} = y$$

$$47.5 \text{ pounds} = y$$

Therefore, Karen needs **47.5** pounds of flour.

7. First, calculate the total number of minutes Melissa spent on her cell phone in January.

$$\frac{25 \text{ minutes}}{1 \text{ day}} \times 31 \text{ days} = 775 \text{ minutes}$$

Next, calculate her cost per minute.

$$\frac{\$108.50}{775 \text{ minutes}} = \$0.14 \text{ per minute}$$

Therefore, Melissa pays **$0.14** per minute.

8. The volume of a solid is measured in cubic units.

With the way the formula is written, the calculated volume of the cone would be in square units.

$$\begin{aligned} V &= \tfrac{1}{3}\pi(r \text{ units})(h \text{ units}) \\ &= \left(\tfrac{1}{3}\pi r h\right) \text{square units} \end{aligned}$$

Therefore, James could tell there was an error because **the calculated volume would be in square units instead of cubic units**.

9. Substitute the values given for π, L, σ, and $t$into the equation, and solve for r.

$$\begin{aligned} L &= 4\pi r^2 \sigma t^4 \\ 2.532 \times 10^{21} \text{ W} &= 4(3.14)r^2\left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}\right)(3{,}000 \text{ K})^4 \\ 2.532 \times 10^{21} \text{ W} &= 12.56r^2\left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}\right)\left(8.1 \times 10^{13} \text{ K}^4\right) \\ 2.532 \times 10^{21} \text{ W} &= 12.56r^2\left(4.5927 \times 10^{6} \frac{\text{W}}{\text{m}^2}\right) \\ 2.532 \times 10^{21} \text{ W} &= \left(5.76843 \times 10^{7} \frac{\text{W}}{\text{m}^2}\right)r^2 \\ \frac{2.532 \times 10^{21} \text{ W}}{\left(5.76843 \times 10^{7} \frac{\text{W}}{\text{m}^2}\right)} &= r^2 \\ 4.38941 \times 10^{13} \text{ m}^2 &= r^2 \\ 6.62526 \times 10^{6} \text{ m} &= r \end{aligned}$$

Therefore, the approximate radius of the star is **6.62526×10^6 m**.

10. Substitute the given values for the mass and the velocity into the kinetic energy formula.

$$
\begin{aligned}
E_k &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(50\ \text{kg})\left(22\ \frac{\text{m}}{\text{s}}\right)^2 \\
&= 25\ \text{kg}\left(484\ \frac{\text{m}^2}{\text{s}^2}\right) \\
&= 12{,}100\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}
\end{aligned}
$$

So, $12{,}100\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$ is the kinetic energy of a 50 kg mass traveling at 22 meters per second.

Algebra I: Create Systems of Equations and Inequalities

1. Tara is thinking of two numbers. The first number is four more than twice the second number. The sum of the two numbers is 15. Which system of equations can be used to determine the first number, *x*, and the second number, *y*?

A. $x + 15 = y$
$y = 2 + 4x$

B. $x + y = 4$
$x = 15 + 2y$

C. $x + y = 15$
$x = 4 + 2y$

D. $x + y = 15$
$x + 2y = 4$

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2. Peter sold a box of 24 books at a yard sale for a total of $53.50. He sold the paperback books for $1.70 each and sold the hardcover books for $2.97 each. Which system of equations can be used to determine the number of $1.70 paperback books, *x*, and the number of $2.97 hardcover books, *y*, that were sold at the yard sale?

A. $x + y = 24$
$1.7x + 2.97y = 53.5$

B. $2.97x - 1.7y = 24$
$x + y = 53.5$

C. $y = x - 53.5$
$1.7x + 2.97y = 24$

D. $x + y = 24$
$2.97y = -1.7x + 53.5$

3. Wayne has a total of 200 Major League Baseball cards from both the American League and the National League. The number of American League cards is 5 more than four times the number of National League cards. Which system of equations can be used to find how many American Leagues cards, A, and National League cards, N, Wayne has?

A.
$A + N = 200$
$N = 4A - 5$

B.
$A = 4N$
$A - 200 = N$

C.
$A - N = 200$
$N = 4A + 5$

D.
$A + N = 200$
$A = 4N + 5$

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4. Mr. Wilson's class took an exam consisting of multiple choice and fill-in-the-blank questions. It takes Katie 3 minutes to answer a multiple choice question and 4 minutes to answer a fill-in-the-blank question. Katie finished the exam in 160 minutes. It takes Robert 4 minutes to answer a multiple choice question and 6 minutes to answer a fill-in-the-blank question. Robert finished the exam in 230 minutes. Which system of equations below can be used to determine the number of multiple choice and fill-in-the-blank questions that were on the exam?

(Let x represent the number of multiple choice questions and y represent the number of fill-in-the-blank questions.)

A.
$6x + 4y = 230$
$4x + 3y = 160$

B.
$3x + 4x = 160$
$4y + 6y = 230$

C.
$3x + 4y = 230$
$4x + 6y = 160$

D.
$3x + 4y = 160$
$4x + 6y = 230$

5. Elizabeth, Shirley, and Sharon are carrying some snacks for a field trip.

Elizabeth has taken 3 packets of nuts, 2 packets of pretzels, and 3 packets of raisin cookies. These snacks cost Elizabeth $8.65.
Shirley has taken 4 packets of nuts, 2 packets of pretzels, and 1 packet of raisin cookies. These snacks cost Shirley $10.75.
Sharon has taken 2 packets of nuts, 5 packets of pretzels, and 2 packets of raisin cookies. These snacks cost Elizabeth $11.57.

Which of the following systems of equations can be used to find the cost of 1 packet of nuts, *n*, 1 packet of pretzels, *p*, and 1 packet of raisin cookies, *s*?

○ **A.** $$\begin{cases} 3n + 2p + 3s = 8.65 \\ 4n + 2p + s = 10.75 \\ 2n + 5p + 2s = 11.57 \end{cases}$$

○ **B.** $$\begin{cases} 2n + 3p + 2s = 8.65 \\ 2n + 2p + s = 10.75 \\ 2n + 5p + 3s = 11.57 \end{cases}$$

○ **C.** $$\begin{cases} 2n + 2p + 3s = 8.65 \\ 2n + 4p + s = 10.75 \\ 3n + 5p + 2s = 11.57 \end{cases}$$

○ **D.** $$\begin{cases} 3n + 2p + 2s = 8.65 \\ 4n + 2p + s = 10.75 \\ 2n + 5p + 5s = 11.57 \end{cases}$$

6. A library owner wants to purchase books for the library. He wants to purchase a maximum of 27 books between short stories and novels. The cost of each short-story book is $6 and the cost of each novel is $10.

If he cannot spend more than $90 on books, which system of inequalities below can be used to determine the number of short-story books, *s*, and the number of novels, *n*, he can purchase?

- **A.** $s - n \leq 27$
 $\$6.00s + \$10.00n \geq \$90$
- **B.** $s - n \leq 27$
 $\$6.00s + \$10.00n \leq \$90$
- **C.** $s + n \leq 27$
 $\$6.00s + \$10.00n \leq \$90$
- **D.** $s + n \leq 27$
 $\$6.00s - \$10.00n \geq \$90$

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7. At the baseball game, 3 hot dogs and 6 soft drinks cost $27, and 4 hot dogs and 3 soft drinks cost $21. Which system of equations below can be used to determine the price of a hot dog and the price of a soft drink?

(Let *x* represent the cost of a hot dog and *y* represent the cost of a soft drink.)

- **A.** $3x + 4y = 27$
 $6x + 3y = 21$
- **B.** $7x + 9y = 48$
- **C.** $3x + 6y = 27$
 $4x + 3y = 21$
- **D.** $3x + 4x = 21$
 $6y + 3y = 27$

8. At the motocross races, four hamburgers and six soft drinks cost $30, and five hamburgers and three soft drinks cost $24. Which system of linear equations below can be used to determine the price of a hamburger and the price of a soft drink?

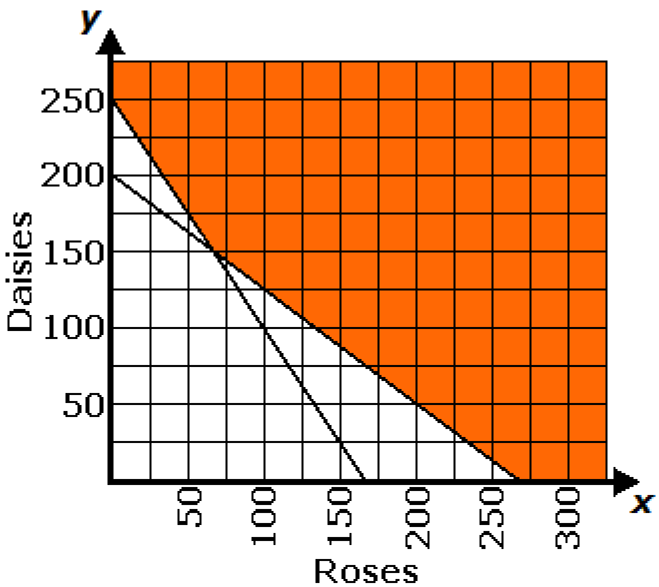
Let x represent the cost of a hamburger, and let y represent the cost of a soft drink.

- **A.** $9x + 9y = 54$
- **B.** $4x + 5y = 30$
  $6x + 3y = 24$
- **C.** $4x + 6y = 30$
  $5x + 3y = 24$
- **D.** $4x + 5x = 27$
  $6y + 3y = 27$

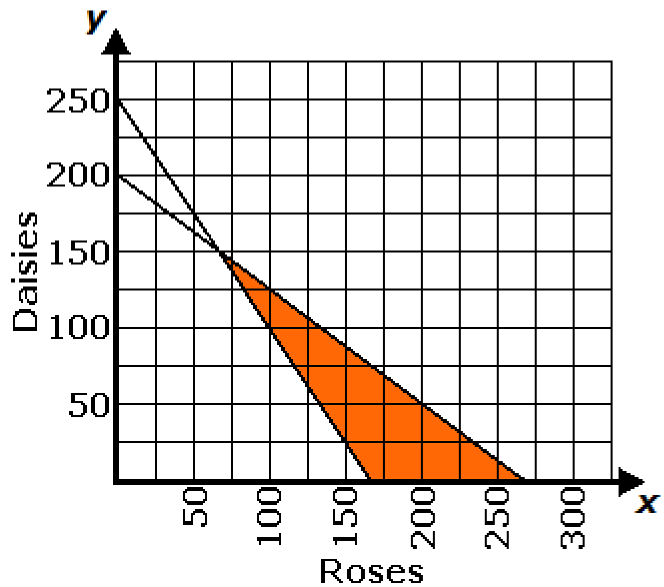
9.

Martha is planting a garden that will cover up to 400 square feet. She wants to plant two types of flowers, daisies and roses. Each daisy covers 2 square feet and each rose covers 1.5 square feet. Daisies cost $2 a piece and each rose costs $3 a piece. Martha doesn't want to spend over $500 on her garden.

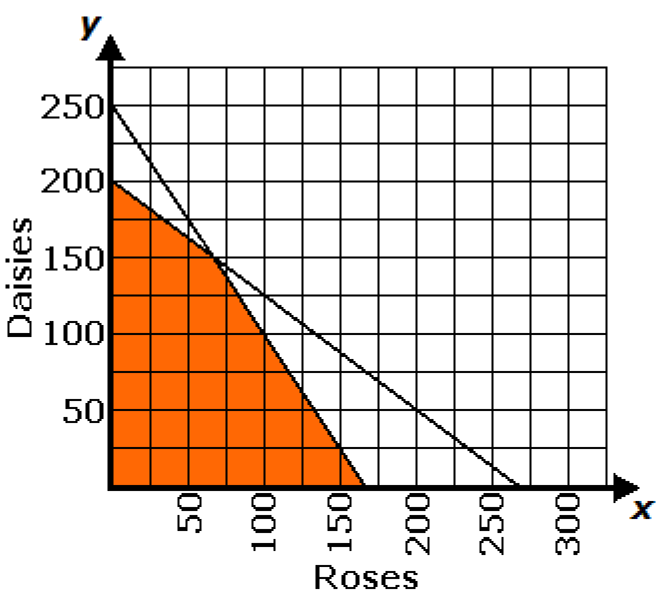
If the solution region is the amount of each type of flower Martha can plant, which graph's shaded area represents the solution region?



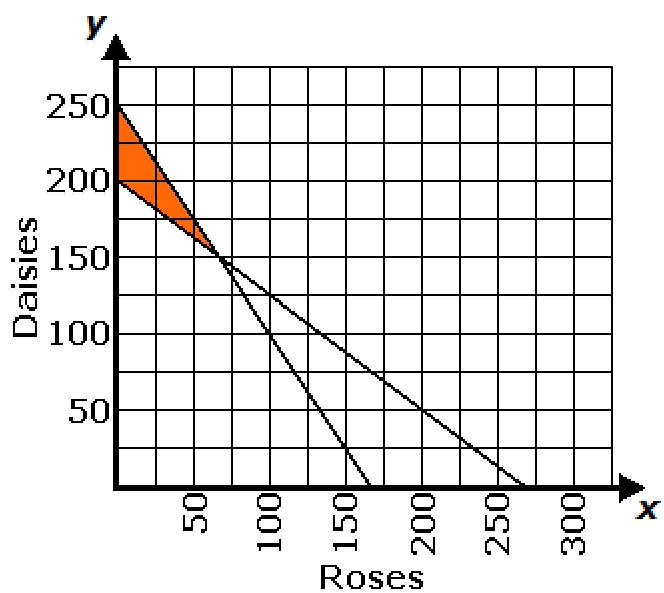
W.



X.



Y.



Z.

- **A.** Y
- **B.** Z
- **C.** X
- **D.** W

10. The school wants to buy a new speaker system for the football stadium with the money they raised from the bake sale. They compared speaker system prices from two different distributors.

From Schools'R'Us, a system, *t*, costs $210 less than twice the price of the same speaker system, *d*, at The Warehouse. The difference in price between the speaker system at Schools'R'Us and The Warehouse is $192.

Which system of equations can be used to determine the price of the speaker system at each distributor?

A. $2t - d = -210$
$t - d = 192$

B. $t - 2d = -210$
$-t + d = 192$

C. $t - 2d = -210$
$t - d = 192$

D. $2t - d = -210$
$-t + d = 192$

Answers: Create Systems of Equations & Inequalities

1. C
2. A
3. D
4. D
5. A
6. C
7. C
8. C
9. A
10. C

Explanations

1. The problem states that x is the first number and y is the second number. Since the first number is four more than twice the second number, the first equation becomes the following.

$$x = 4 + 2y$$

It also states that the two numbers add up to 15, so the second equation is the following.

$$x + y = 15$$

Thus, the system of equations that matches the given situation is shown below.

$$\mathbf{x + y = 15}$$
$$\mathbf{x = 4 + 2y}$$

2. The problem states that x is the number of $1.70 paperback books and y is the number of $2.97 hardcover books. It also states that there are 24 books altogether, so the first equation is the following.

$$x + y = 24$$

Since the total sale from the books is $53.50, the second equation is the following.

$$\$1.70x + \$2.97y = \$53.50$$

Thus, the system of equations that matches the given situation is below.

$$\mathbf{x + y = 24}$$
$$\mathbf{1.7x + 2.97y = 53.5}$$

3. The total number of American League cards, A, and National League cards, N, is 200, as shown in the equation below.

$$A + N = 200$$

The number of American League cards is 5 more than four times the number of National League cards, or A is 5 more than $4N$, as shown in the equation below.

$$A = 4N + 5$$

The two equations together give the following system of equations.

$$A + N = 200$$
$$A = 4N + 5$$

4. Katie took 3 minutes per multiple choice question and 4 minutes per fill-in-the-blank question for a total time of 160 minutes, as shown in the equation below.

$$3x + 4y = 160$$

Robert took 4 minutes per multiple choice question and 6 minutes per fill-in-the-blank question for a total time of 230 minutes, as shown in the equation below.

$$4x + 6y = 230$$

The two equations together give the following system of equations.

5. To set up the system of equations, translate each sentence.

It is given that n equals the cost of 1 packet of nuts, p equals the cost of 1 packet of pretzels, and s equals the cost of 1 packet of raisin cookies.

Elizabeth's snacks cost $8.65. She took 3 packets of nuts, 2 packets of pretzels, and 3 packets of raisin cookies.

$$3n + 2p + 3s = 8.65$$

Shirley's snacks cost $10.75. She took 4 packets of nuts, 2 packets of pretzels, and 1 packet of raisin cookies.

$$4n + 2p + s = 10.75$$

Sharon's snacks cost $11.57. She took 2 packets of nuts, 5 packets of pretzels, and 2 packets of raisin cookies.

$$2n + 5p + 2s = 11.57$$

Combining all three of these equations gives the following system of equations.

$$\begin{cases} 3n + 2p + 3s = 8.65 \\ 4n + 2p + s = 10.75 \\ 2n + 5p + 2s = 11.57 \end{cases}$$

6. Since the library owner can purchase a maximum of 27 books, use the following inequality to represent the number of short-story books and novels.

$$s + n \leq 27$$

Each short-story book costs $6.00 and each novel costs $10.00. He can spend no more than $90 on books. The inequality below represents the situation.

$$\$6.00s + \$10.00n \leq \$90$$

Combining these two inequalities together creates the system.

$$s + n \leq 27$$

$$6.00s + \$10.00n \leq \$90$$

7. Let *x* represent the cost of a hot dog and *y* represent the cost of a soft drink.

Write an equation to represent "3 hot dogs and 6 soft drinks cost $27."

$$3x + 6y = 27$$

Write an equation to represent "4 hot dogs and 3 soft drinks cost $21."

$$4x + 3y = 21$$

Since *x* and *y* need to satisfy both sets of equations, write the equations together as a system of equations.

$$3x + 6y = 27$$
$$4x + 3y = 21$$

8. Since four hamburgers and six soft drinks cost $30, the equation below can be produced.

$$4x + 6y = 30$$

Similarly, five hamburgers and three soft drinks, which cost $24, produces the equation below.

$$5x + 3y = 24$$

Since *x* and *y* need to satisfy both sets of equations, write both equations together as a system of equations.

$$4x + 6y = 30$$
$$5x + 3y = 24$$

9. First, form the inequalities needed for the graph using *x* for roses and *y* for daisies.

Since each rose covers 1.5 square feet, each daisy covers 2 square feet, and the garden covers up to 400 square feet, use the following inequality.

$$1.5x + 2y \leq 400$$

Since each rose costs $3 a piece, each daisy costs $2 a piece, and Martha cannot spend more than $500, use the following inequality.

$$3x + 2y \leq 500$$

Thus, the solution region can be found using this system of inequalities.

$$1.5x + 2y \leq 400$$
$$3x + 2y \leq 500$$

In order to graph the inequalities, convert each equation into slope-intercept form.

$$1.5x + 2y \leq 400$$
$$1.5x - 1.5x + 2y \leq 400 - 1.5x$$
$$2y \leq -1.5x + 400$$
$$\frac{2y}{2} \leq \frac{-1.5x + 400}{2}$$
$$y \leq -\frac{3}{4}x + 200$$

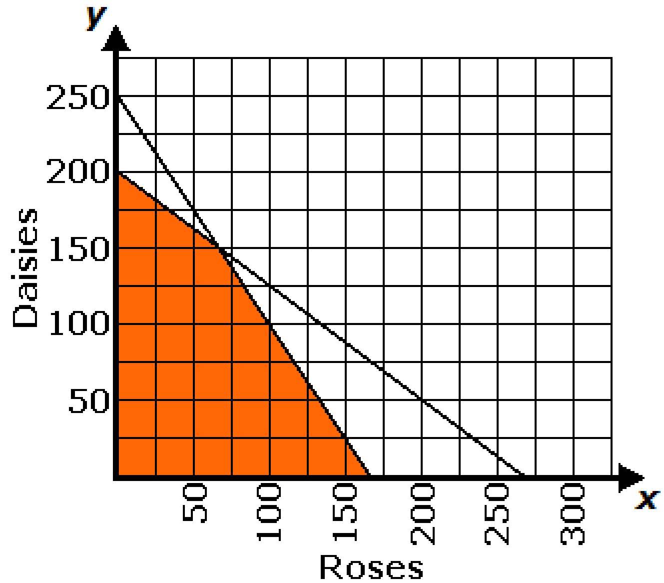
$$3x + 2y \leq 500$$
$$3x - 3x + 2y \leq 500 - 3x$$
$$2y \leq -3x + 500$$
$$\frac{2y}{2} \leq \frac{-3x + 500}{2}$$
$$y \leq -\frac{3}{2}x + 250$$

Since both equations are allowed to be equal, graph them using solid lines.

The equation on the left shows *y* is less than or equal to the right side, so the solution region will be below this line.

The equation on the right shows *y* is less than or equal to the right side, so the solution region will be below this line.

The region that is below the first line and below the second line forms the solution region, which is shaded as shown below.



10. The price at Schools'R'Us, *t*, is $210 less than twice the price at The Warehouse, *d*.

$$2d - 210 = t$$
$$-210 = t - 2d$$
$$t - 2d = -210$$

The difference between the price at School'R'Us, *t*, and the price at The Warehouse, *d*, is $192.

$$t - d = 192$$

Since *t* and *d* need to satisfy both sets of equations, write the equations together as a system of equations.

$$\mathbf{t - 2d = -210}$$
$$\mathbf{t - d = 192}$$

Algebra I: Rewrite Linear Variable Equations

1. Electrical energy (*E*) is measured in kilowatt hours (kwh) using the formula $E = P \cdot t$, where *P* is the power in kilowatts and *t* is the time in hours.

If 112 kwh of electrical energy are created over a period of 16 hours, how many kilowatts of power are used each hour?

- **A.** 1,792
- **B.** 128
- **C.** 96
- **D.** 7

---

2. Given the following formula, solve for *h*.

$$A = \frac{1}{2}(b_1 + b_2)h$$

- **A.** $h = \frac{2A}{b_1 + b_2}$
- **B.** $h = \frac{A}{2(b_1 + b_2)}$
- **C.** $h = \frac{b_1 + b_2}{2A}$
- **D.** $h = 2A - (b_1 + b_2)$

---

3. Given the following formula, solve for *k*.

$$F = \frac{kq_1q_2}{r^2}$$

- **A.** $k = \frac{Fq_1q_2}{r^2}$
- **B.** $k = \frac{q_1q_2r^2}{F}$
- **C.** $k = Fr^2 - q_1q_2$
- **D.** $k = \frac{Fr^2}{q_1q_2}$

4. Given the following formula, solve for *m*.

$$y = mx + b$$

- **A.** $m = \frac{y + b}{x}$
- **B.** $m = x(y + b)$
- **C.** $m = \frac{y - b}{x}$
- **D.** $m = x(y - b)$

---

5. The volume for a rectangular prism is given by the formula $V = l \cdot w \cdot h$, where *l* is the length of the prism, *w* is the width of the prism, and *h* is the height of the prism.

The volume of a rectangular prism is 343 cubic inches. If the length and height are 7 inches each, what is the width of the rectangular prism?

- **A.** 42 inches
- **B.** 329 inches
- **C.** 7 inches
- **D.** 49 inches

---

6. Given the following formula, solve for *y*.

$$w = \frac{x - y}{2} - z$$

- **A.** $y = x - (2w + z)$
- **B.** $y = 2(w + z) - x$
- **C.** $y = 2w + z - x$
- **D.** $y = x - 2(w + z)$

7. The volume for a rectangular prism is given by the formula $V = l \cdot w \cdot h$, where l is the length of the prism, w is the width of the prism, and h is the height of the prism.

If the volume of a rectangular prism with a length and width of 3 inches is 72 cubic inches, what is the height of the rectangular prism?

- **A.** 66 inches
- **B.** 12 inches
- **C.** 8 inches
- **D.** 24 inches

---

8. Given the following formula, solve for a.

$$s = \frac{a + b + c}{2}$$

- **A.** $a = s - \frac{b + c}{2}$
- **B.** $a = 2s - b + c$
- **C.** $a = s - \frac{b - c}{2}$
- **D.** $a = 2s - b - c$

---

9. Given the following formula, solve for c.

$$3a - \frac{b}{c - 1} = 2a$$

- **A.** $c = \frac{b}{5a} - 1$
- **B.** $c = \frac{a + b}{a}$
- **C.** $c = \frac{5a + b}{5a}$
- **D.** $c = \frac{a}{b} + 1$

---

10. Given the following formula, solve for y_1.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- **A.** $y_1 = m(x_2 - x_1) - y_2$
- **B.** $y_1 = y_2 - m(x_2 - x_1)$
- **C.** $y_1 = mx_2 - x_1 - y_2$
- **D.** $y_1 = y_2 - mx_2 - x_1$

Answers: Rewrite Linear Variable Equations

1. D
2. A
3. D
4. C
5. C
6. D
7. C
8. D
9. B
10. B

Explanations

1. Use the formula $E = P \cdot t$ and the values E = 112 kwh and t = 16 hours. Substitute these values into the formula and solve for P.

$$E = P \cdot t$$

$$\frac{E}{t} = P$$

$$P = \frac{E}{t}$$

$$P = \frac{112 \text{ kwh}}{16 \text{ hours}}$$

$$P = 7 \text{ kilowatts}$$

2. Use algebra and simplification to solve the equation for h.

$$\begin{aligned} A &= \tfrac{1}{2}(b_1 + b_2)h \\ 2A &= (b_1 + b_2)h \\ \frac{2A}{b_1 + b_2} &= h \end{aligned}$$

3. Use algebra and simplification to solve the equation for k.

$$\begin{aligned} F &= \frac{kq_1q_2}{r^2} \\ Fr^2 &= kq_1q_2 \\ \frac{Fr^2}{q_1q_2} &= k \end{aligned}$$

4. Use algebra and simplification to solve the equation for *m*.

$$y = mx + b$$

$$y - b = mx$$

$$\frac{y - b}{x} = m$$

5. Use the formula $V = l \cdot w \cdot h$ and the values $V = 343$ cubic inches and $l = w = h$. Solve for *w* and substitute these values into the formula.

$$V = l \cdot w \cdot h$$

$$\frac{V}{l \cdot h} = w$$

$$w = \frac{V}{l \cdot h}$$

$$w = \frac{343 \text{ cubic inches}}{7 \text{ inches} \cdot 7 \text{ inches}}$$

$$w = 7 \text{ inches}$$

6. Use algebra and simplification to solve the equation for *y*.

$$w = \frac{x - y}{2} - z$$

$$w + z = \frac{x - y}{2}$$

$$2(w + z) = x - y$$

$$2(w + z) + y = x$$

$$y = x - 2(w + z)$$

7. Use the formula $V = l \cdot w \cdot h$ and the values $V = 72$ cubic inches and $l = w = 3$ inches. Solve for *h* and substitute these values into the formula.

$$V = l \cdot w \cdot h$$

$$\frac{V}{l \cdot w} = h$$

$$h = \frac{V}{l \cdot w}$$

$$h = \frac{72 \text{ cubic inches}}{3 \text{ inches} \cdot 3 \text{ inches}}$$

$$h = 8 \text{ inches}$$

8. Use algebra and simplification to solve the equation for *a*.

$$s = \frac{a + b + c}{2}$$

$$2s = a + b + c$$

$$2s - b - c = a$$

9. Use algebra and simplification to solve the equation for *c*.

$$3a - \frac{b}{c - 1} = 2a$$

$$a = \frac{b}{c - 1}$$

$$c - 1 = \frac{b}{a}$$

$$c = 1 + \frac{b}{a}$$

$$c = \frac{a}{a} + \frac{b}{a}$$

$$c = \frac{a + b}{a}$$

10. Use algebra and simplification to solve the equation for y_1.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m(x_2 - x_1) &= y_2 - y_1 \\ m(x_2 - x_1) - y_2 &= -y_1 \\ -1[m(x_2 - x_1) - y_2] &= y_1 \\ -m(x_2 - x_1) + y_2 &= y_1 \\ y_2 - m(x_2 - x_1) &= y_1 \end{aligned}$$

Algebra I: Rational Exponents

1. $2^{\frac{1}{2}} = ?$

- ◯ **A.** $\frac{1}{2^2}$
- ◯ **B.** 1
- ◯ **C.** $\sqrt{2}$
- ◯ **D.** $\left(\frac{1}{2}\right)^2$

---

2. $\sqrt[3]{3} \cdot 2$

Which of the following is equal to the expression above?

- ◯ **A.** $216^{\frac{1}{3}}$
- ◯ **B.** $54^{\frac{1}{3}}$
- ◯ **C.** $24^{\frac{1}{3}}$
- ◯ **D.** $6^{\frac{1}{3}}$

---

3. $3^{\frac{1}{4}} + 6^3$

Which of the following is equivalent to the expression above?

- ◯ **A.** $\sqrt[4]{3} + 18$
- ◯ **B.** $\sqrt[3]{4} + 216$
- ◯ **C.** $\sqrt[4]{3} + 216$
- ◯ **D.** $\frac{1}{3^4} + 216$

4. Simplify.

$$81^{\frac{1}{4}}$$

A. 3

B. 9

C. $\frac{1}{324}$

D. $\frac{81}{4}$

---

5. Rewrite the following.

$$x^{\frac{2}{3}}$$

A. $\sqrt[2]{x^3}$

B. $\left(\frac{1}{x^3}\right)^2$

C. $\frac{x^2}{x^3}$

D. $\sqrt[3]{x^2}$

---

6. Which of the following shows that $\sqrt[4]{5} = 5^{\frac{1}{4}}$?

A. $\left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5^{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}} = 5^{\frac{4}{4}} = 5^1 = 5$

B. $\left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5 \cdot \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) = 5 \cdot \frac{4}{4} = 5$

C. $\left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 4 \cdot 5^{\frac{1}{4}} = 4 \cdot \frac{1}{4} \cdot 5 = 5$

D. $\left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5^{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = 5^{\frac{4}{4}} = 5^1 = 5$

7. Simplify the following expression.

$$2^{\frac{1}{3}} \div 2^{\frac{7}{3}}$$

- A. $\frac{1}{2}$
- B. $\frac{1}{8}$
- C. $\frac{1}{4}$
- D. 4

---

8. $$4^4 - 3^{\frac{1}{5}}$$

Which of the following is equivalent to the expression above?

- A. $256 - \sqrt[5]{3}$
- B. $256 - \sqrt[3]{5}$
- C. $16 - \sqrt[5]{3}$
- D. $256 - \frac{1}{3^5}$

---

9. Which of the following is equivalent to $7^{-\frac{1}{6}}$?

- A. $-\sqrt[6]{7}$
- B. $\frac{1}{\sqrt{7^6}}$
- C. $-\sqrt{7^6}$
- D. $\frac{1}{\sqrt[6]{7}}$

10. Which of the following is equivalent to $23^{-\frac{1}{2}}$?

- **A.** $-\sqrt{23}$
- **B.** -23
- **C.** $\frac{1}{\sqrt{23}}$
- **D.** $-\frac{1}{23}$

Answers: Rational Exponents

1. C
2. C
3. C
4. A
5. D
6. D
7. C
8. A
9. D
10. C

Explanations

Recall the following rule.

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

For $2^{\frac{1}{2}}, a = 2$ and $n = 2$.

1. Therefore, $2^{\frac{1}{2}} = \sqrt{2}$.

2.

$$(ab)^n = a^n \cdot b^n$$
$$\sqrt[n]{b} = b^{\frac{1}{n}} \text{ with } n \neq 0$$

$$\begin{aligned}\sqrt[3]{3} \cdot 2 &= 3^{\frac{1}{3}} \cdot 2 \\ &= 3^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} \\ &= (3 \cdot 8)^{\frac{1}{3}} \\ &= 24^{\frac{1}{3}}\end{aligned}$$

3. Use the following rule of exponents to simplify.

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$3^{\frac{1}{4}} + 6^3 = \sqrt[4]{3} + 216$$

Use the laws of exponents to simplify the expression.

$$81^{\frac{1}{4}} = \sqrt[4]{81}$$
$$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3}$$
$$= 3$$

4.

Use the laws of exponents to rewrite the expression.

$$x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

5.

Since $\left(\sqrt[4]{5}\right)^4 = \sqrt[4]{5} \cdot \sqrt[4]{5} \cdot \sqrt[4]{5} \cdot \sqrt[4]{5} = 5$, it is sufficient to show that $\left(5^{\frac{1}{4}}\right)^4 = 5$ in order to show that $\sqrt[4]{5} = 5^{\frac{1}{4}}$.

Remember the following exponent rule with the nonzero real number a and the integers m and n.

$$a^m \cdot a^n = a^{m+n}$$

Thus, the following is true.

$$\left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = 5^{\frac{4}{4}} = 5^1 = 5$$

6.

7. When dividing two terms with the same base, subtract the exponents.

$$a^m \div a^n = a^{(m-n)}$$
$$2^{\frac{1}{3}} \div 2^{\frac{7}{3}} = 2^{\left(\frac{1}{3} - \frac{7}{3}\right)}$$
$$= 2^{(-2)}$$
$$= \frac{1}{2^2}$$
$$= \frac{1}{4}$$

8. Use the following rule of exponents to simplify.

$$\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

$$4^4 - 3^{\frac{1}{5}} = 256 - \sqrt[5]{3}$$

Apply the following laws of exponents.

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$
$$x^{-m} = \frac{1}{x^m}$$

$$7^{-\frac{1}{6}} = \frac{1}{7^{\frac{1}{6}}}$$
$$= \frac{1}{\sqrt[6]{7}}$$

9.

10. Apply the following laws of exponents.

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$
$$x^{-m} = \frac{1}{x^m}$$

$$23^{-\frac{1}{2}} = \frac{1}{23^{\frac{1}{2}}}$$

$$23^{-\frac{1}{2}} = \frac{1}{\sqrt{23}}$$

Algebra I: Evaluate Functions

1. Digital Genius manufactures a cell phone charging pad that can fully charge any cell phone that is placed on it in 15 minutes. The analytics department determined that at a price of $115.00 each, the demand would be 3 thousand pads, and at $75.00 each, the demand would be 13 thousand pads. Based on the assumption that a linear relationship exists between price and demand, the price-demand equation is given by the following function.

$$p(x) = -4x + 127$$

In this function, *x* is the number of thousands of pads in demand. At what price point should they sell the pads to have a demand of 11 thousand pads?

- **A.** $31.75
- **B.** $83.00
- **C.** $29.00
- **D.** $44.00

---

2. Find the approximate value of *h*(-1) for the function below.

$$h(x) = 22e^{x} + 53$$

- **A.** 75.37
- **B.** 53.37
- **C.** 61.09
- **D.** 8.09

---

3. Find the value of *g*(5) for the function below.

$$g(x) = 36 \cdot \left(\frac{1}{3}\right)^{x}$$

- **A.** $\frac{4}{27}$
- **B.** $\frac{1}{243}$
- **C.** 248,832
- **D.** 8,748

4. Find the approximate value of *g*(4) for the function below.

$$g(x) = e^{x} - 128$$

- **A.** -73.4
- **B.** 54.6
- **C.** -84.69
- **D.** 4.88

---

5. Find the value of *h*(3) for the function below.

$$h(x) = 3 \cdot 3^{(2x - 3)} + 15$$

- **A.** 13,824
- **B.** 126
- **C.** 2,202
- **D.** 96

---

6. The N71-90 virus will give an infected person a mild rash on their arms once the person has 1 billion of the virus cells in their body. After an initial infection with just one cell, each virus cell will divide into two cells every four hours. After 30 divisions there are 1,073,741,824 cells and the rash starts. Once the rash starts, the immune system kicks in and kills all of the virus cells within an hour. The number of virus cells in the body can be determined using the function below.

$$V(h) = 2^{\frac{h}{4}}$$

In this function, *h* is the number of hours after infection. If Jolene is infected by one virus cell, how many virus cells will she have after 28 divisions?

- **A.** 256
- **B.** 268,435,456
- **C.** 128
- **D.** 536,870,912

---

7. Joe-Elk tractors offers a lease/buy program that will allow a farmer to lease a new C7060 tractor for eight years with an option to purchase for a specified value at any time up to the end of the lease. The purchase price of the tractor is determined by the function below.

$$P(m) = 240{,}300(0.9925)^{96 - m}$$

In this function, *m* is the number of months remaining in the lease. Thirty-six months into the lease, Jacob is considering purchasing the tractor. What is the purchase price of the tractor?

- **A.** $183,253.52
- **B.** $116,613.60
- **C.** $116,589.60
- **D.** $152,962.40

---

8. Find the value of *f*(-2) for the function below.

$$f(x) = 0.5^{(x - 2)}$$

- **A.** 4
- **B.** 1
- **C.** 16
- **D.** 0.5

---

9. Following market trends, FireCore started making 10" tablet computers. For this part of their operation, they have fixed costs of $50,500.00 per month and variable costs of $144.90 per tablet made. The company sells the tablets for $189.00 each. The company's profit function for the tablets can be found using the following function.

$$p(x) = 44.1x - 50{,}500$$

In this function, *x* is the number of tablets sold. After their debut marketing blitz, the company sold 2,220 tablets during the first month. How much did FireCore make from their tablet division for the first month?

- **A.** $47,402.00
- **B.** $42,992.00
- **C.** $419,580.00
- **D.** $97,902.00

---

10. Find the value of *f*(40) for the function below.

$$f(x) = \frac{1}{2}x + 130$$

- **A.** -110
- **B.** 170
- **C.** 150
- **D.** 20

Answers: Evaluate Functions

1. B
2. C
3. A
4. A
5. D
6. B
7. A
8. C
9. A
10. C

Explanations

1. Use the price-demand equation to determine the price point that will correlate to a demand of 11 thousand pads.

$$\begin{aligned} p(11) &= -4(11) + 127 \\ &= -44 + 127 \\ &= 83 \end{aligned}$$

To create a demand of 11 thousand pads, the price of a pad should be set at **$83.00**.

2. First, substitute -1 for *x* in the given function. Then, evaluate.

$$h(x) = 22e^{x} + 53$$

$$\begin{aligned} h(-1) &= 22e^{(-1)} + 53 \\ &\approx 61.09 \end{aligned}$$

3. First, substitute 5 for *x* in the given function. Then, evaluate.

$$g(x) = 36 \cdot \left(\frac{1}{3}\right)^{x}$$

$$\begin{aligned} g(5) &= 36 \cdot \left(\frac{1}{3}\right)^{5} \\ &= 36 \cdot \frac{1}{243} \\ &= \frac{4}{27} \end{aligned}$$

4. First, substitute 4 for *x* in the given function. Then, evaluate.

$$g(x) = e^{x} - 128$$

$$\begin{aligned} g(4) &= e^{4} - 128 \\ &\approx -73.4 \end{aligned}$$

5. First, substitute 3 for *x* in the given function. Then, evaluate.

$$h(x) = 3 \cdot 3^{(2x - 3)} + 15$$

$$\begin{aligned} h(3) &= 3 \cdot 3^{[2(3) - 3]} + 15 \\ &= 3 \cdot 3^{3} + 15 \\ &= 3 \cdot 27 + 15 \\ &= 96 \end{aligned}$$

6. Since the function is based on hours of infection instead of number of divisions, first find the number of hours that must pass to have 28 divisions. Since cell division takes place every four hours, Jolene must have had the infection for

$$28 \text{ divisions} \times 4 \frac{\text{hours}}{\text{division}} = 112 \text{ hours.}$$

Use the virus cells function to determine the number of virus cells Jolene will have after 112 hours.

$$\begin{aligned} V(112) &= 2^{\frac{(112)}{4}} \\ &= 2^{28} \\ &= 268{,}435{,}456 \end{aligned}$$

After 28 divisions, or 112 hours, Jolene will have **268,435,456** virus cells.

7. The purchase price function is based on the number of months remaining in the lease. Since the original lease was for eight years and Jacob is only thirty-six months into the lease, there are 96 - 36 = 60 months remaining in the lease. So, use the function to determine the purchase price with sixty months remaining in the lease.

$$\begin{aligned} P(60) &= 240{,}300(0.9925)^{96 - (60)} \\ &= 240{,}300(0.9925)^{36} \\ &= 240{,}300(0.762603081223) \\ &\approx 183{,}253.52 \end{aligned}$$

At thirty-six months into the lease, the purchase price of the tractor is **$183,253.52**.

8. First, substitute -2 for *x* in the given function. Then, evaluate.

$$f(x) = 0.5^{(x\ -\ 2)}$$

$$\begin{aligned} f(-2) &= 0.5^{(-2\ -\ 2)} \\ &= 0.5^{(-4)} \\ &= 16 \end{aligned}$$

9. Use the monthly profit function for the tablets to determine the company's profit for selling 2,220 tablets in a month.

$$\begin{aligned} p(2{,}220) &= 44.1(2{,}220)\ -\ 50{,}500 \\ &= 97{,}902\ -\ 50{,}500 \\ &= 47{,}402 \end{aligned}$$

FireCore made a one month profit of **$47,402.00**.

10. First, substitute 40 for *x* in the given function. Then, evaluate.

$$f(x) = \tfrac{1}{2}x\ +\ 130$$

$$\begin{aligned} f(40) &= \tfrac{1}{2}(40)\ +\ 130 \\ &= \frac{40}{2} + 130 \\ &= 20\ +\ 130 \\ &= 150 \end{aligned}$$

Algebra I: Sequences

1. The recursive formula for an arithmetic sequence is given below. What is the fourth term in the sequence?

$$a_1 = 0$$
$$a_n = -6 + a_{(n-1)}$$

- **A.** -30
- **B.** -12
- **C.** -18
- **D.** -24

---

2. Express the terms of the following geometric sequence recursively.

6 , 18 , 54 , 162 , 486 , ...

- **A.** $t_1 = 6$ and $t_n = 3(t_{n-1}) - 6$, for $n \geq 2$
- **B.** $t_1 = 6$ and $t_n = 3(t_{n-1})$, for $n \geq 2$
- **C.** $t_1 = 6$ and $t_n = t_{n-1} + 3n$, for $n \geq 2$
- **D.** $t_1 = 6$ and $t_n = t_{n-1} + 12$, for $n \geq 2$

---

3. Express the terms of the following sequence by giving an explicit formula.

$$-\frac{1}{2},\ -1\frac{1}{3},\ -2\frac{1}{6},\ -3,\ . . .$$

- **A.** $a_n = \frac{1}{3} - 1\frac{1}{5}n$, where $n = 1, 2, 3, 4, . . .$
- **B.** $a_n = \frac{1}{3}n$, where $n = 1, 2, 3, 4, . . .$
- **C.** $a_n = \frac{5}{6}n$, where $n = 1, 2, 3, 4, . . .$
- **D.** $a_n = \frac{1}{3} - \frac{5}{6}n$, where $n = 1, 2, 3, 4, . . .$

4. Stacy is saving money for a vacation. She deposits \$72 every month in a non-interest bearing savings account having an initial balance of \$105. Choose the equation below that gives the account balance, B_n, in the n^{th} month. Then, use this equation to determine the account balance in the 9^{th} month.

- **A.** $B_n = \$105 + \$72 \cdot (n - 1)$; $B_9 = \$681$
- **B.** $B_n = \$105 + \$72 \cdot (n)$; $B_9 = \$825$
- **C.** $B_n = \$105 + \$72 \cdot (n)$; $B_9 = \$681$
- **D.** $B_n = \$105 + \$72 \cdot (n - 1)$; $B_9 = \$825$

---

5. What are the first four terms of the sequence defined by the following equation?

$$a_n = 3 \cdot 2^{n-1}$$

- **A.** 3 , 6 , 12 , 24 ...
- **B.** 6 , 12 , 24 , 48 ...
- **C.** 6 , 36 , 216 , 1,296 ...
- **D.** 3 , 6 , 36 , 216 ...

6.
Identify the correct formula for the following sequence.

-9, -6, -3, 0, 3, ...

- **A.** $a_n = -3n + 12$
- **B.** $a_n = 3n - 12$
- **C.** $a_n = -12n + 3$
- **D.** $a_n = 3n + 12$

---

7. Given the recursive formula, $u(1) = 47$ and $u(n + 1) = u(n) + 4$, for $n = 1, 2, 3, \ldots$, find the explicit formula for $u(n)$.

- **A.** $u(n) = 47 + 4n$
- **B.** $u(n) = 43 + 4n$
- **C.** $u(n) = 4 + 43n$
- **D.** $u(n) = 4 + 47n$

8. Express the terms of the following geometric sequence by giving an explicit formula.

$$3 , 15 , 75 , 375 , 1{,}875 , \ldots$$

- **A.** $t_n = 3(5)^n$, where $n = 1, 2, 3, \ldots$
- **B.** $t_n = 5(3)^n$, where $n = 1, 2, 3, \ldots$
- **C.** $t_n = 3(5)^{n-1}$, where $n = 1, 2, 3, \ldots$
- **D.** $t_n = 3(5)(n-1)$, where $n = 1, 2, 3, \ldots$

---

9. Express the terms of the following sequence by giving a recursive formula.

$$8, 16, 24, 32, \ldots$$

- **A.** $a_1 = 8$ and $a_{n+1} = a_n + 8$, where $n = 1, 2, 3, 4, \ldots$
- **B.** $a_1 = 8$ and $a_{n+1} = 8a_n + 8$, where $n = 1, 2, 3, 4, \ldots$
- **C.** $a_1 = 8$ and $a_{n+1} = a_n - 8$, where $n = 1, 2, 3, 4, \ldots$
- **D.** $a_1 = 8$ and $a_{n+1} = 8a_n - 8$, where $n = 1, 2, 3, 4, \ldots$

---

10. Express the terms of the following geometric sequence by giving an explicit formula.

$$4 , 1 , \frac{1}{4} , \frac{1}{16} , \frac{1}{64} , \ldots$$

- **A.** $f(n) = 4\left(\frac{1}{4}\right)^n$, where $n = 1, 2, 3, \ldots$
- **B.** $f(n) = 4\left(\frac{1}{4}\right)(n-1)$, where $n = 1, 2, 3, \ldots$
- **C.** $f(n) = 4\left(\frac{1}{4}\right)^{n-1}$, where $n = 1, 2, 3, \ldots$
- **D.** $f(n) = \frac{1}{4}(4^n)$, where $n = 1, 2, 3, \ldots$

Answers: Sequences

1. C
2. B
3. D
4. A
5. A
6. B
7. B
8. C
9. A
10. C

Explanations

1. In a recursive formula, each term is used to find the next term in the sequence.

Use the recursive formula for the arithmetic sequence to find a_4.

$$
\begin{aligned}
a_1 &= 0 \\
a_2 &= -6 + a_{(2-1)} \\
&= -6 + (0) \\
&= -6 \\
a_3 &= -6 + a_{(3-1)} \\
&= -6 + (-6) \\
&= -12 \\
a_4 &= -6 + a_{(4-1)} \\
&= -6 + (-12) \\
&= -18
\end{aligned}
$$

Therefore, the fourth term in the sequence is **-18**.

2. The recursive formula for a geometric sequence is t_1 = first term, $t_n = r \cdot t_{n-1}$, where r is the common ratio, and $n \geq 2$.

In this sequence, each term (t_n) is the value of the previous term (t_{n-1}) multiplied by 3, the common ratio.

For instance, $t_3 = 3 \cdot t_2 = 3 \cdot 18 = 54$.

Now, write it as a recursive formula.

$$\mathbf{t_1 = 6 \text{ and } t_n = 3(t_{n-1}), \text{ for } n \geq 2}$$

3. The first term of this arithmetic sequence is $a_1 = -\frac{1}{2}$.

Find the common difference in each of the numbers of the sequence. The common difference will always be the absolute value of the difference.

$$-3 - \left(-2\tfrac{1}{6}\right) = -\tfrac{5}{6}$$
$$-2\tfrac{1}{6} - \left(-1\tfrac{1}{3}\right) = -\tfrac{5}{6}$$
$$-1\tfrac{1}{3} - \left(-\tfrac{1}{2}\right) = -\tfrac{5}{6}$$

Each term in the sequence has decreased by $\frac{5}{6}$.

Add $\frac{5}{6}$ to the first term in the sequence to determine the constant of the function.

$$\frac{5}{6} + \left(-\frac{1}{2}\right) = \frac{1}{3}$$

Multiply the common difference of $\frac{5}{6}$ by the number in the sequence, *n*, and subtract it from $\frac{1}{3}$.

n	a_n	Formula	Term
1	a_1	$\frac{1}{3} - \left(\frac{5}{6}\right)1$	$-\frac{1}{2}$
2	a_2	$\frac{1}{3} - \left(\frac{5}{6}\right)2$	$-1\frac{1}{3}$
3	a_3	$\frac{1}{3} - \left(\frac{5}{6}\right)3$	$-2\frac{1}{6}$
n	a_n	$\frac{1}{3} - \left(\frac{5}{6}\right)n$	$\frac{1}{3} - \left(\frac{5}{6}\right)n$

Now, write this as an explicit formula.

$$a_n = \frac{1}{3} - \frac{5}{6}n, \text{ where } n = 1, 2, 3, 4, \ldots$$

4. The situation can be represented by using an arithmetic sequence.

- B_1 = account balance in the 1st month
- B_2 = account balance in the 2nd month
- B_3 = account balance in the 3rd month
- B_n = account balance in the *n*th month

The n^{th} term of an arithmetic sequence with the common difference, d, can be found with the equation below.

$$a_n = a_1 + d \cdot (n - 1)$$

or in this case

$$B_n = B_1 + d \cdot (n - 1)$$

Stacy's account balance in the 1st month, B_1, is equal to $105, and the common difference, d, is equal to the $72 that she deposits every month.

Therefore, the equation $B_n = \$105 + \$72 \cdot (n - 1)$ models the given situation.

Using B_n, $B_9 = \$105 + \$72 \cdot 8 = \$105 + \$576 = \$681$.

Finally, $\mathbf{B_n = \$105 + \$72 \cdot (n - 1)}$ with $\mathbf{B_9 = \$681}$.

5. Use the equation for the n^{th} term of the sequence to find the first four terms.

$$a_1 = 3 \cdot 2^{1-1} = 3 \cdot 2^0 = 3 \cdot 1 = 3$$

$$a_2 = 3 \cdot 2^{2-1} = 3 \cdot 2^1 = 3 \cdot 2 = 6$$

$$a_3 = 3 \cdot 2^{3-1} = 3 \cdot 2^2 = 3 \cdot 4 = 12$$

$$a_4 = 3 \cdot 2^{4-1} = 3 \cdot 2^3 = 3 \cdot 8 = 24$$

⋮

Therefore, the first four terms of the geometric sequence are shown below.

3 , 6 , 12 , 24 ...

6. First, find the difference between consecutive terms in the sequence.

$$-6 - (-9) = 3$$
$$-3 - (-6) = 3$$
$$0 - (-3) = 3$$
$$3 - 0 = 3$$

The difference between consecutive terms in the sequence is constant and equal to 3. Therefore, the formula for the sequence will be in the form $a_n = 3n + C$, where a_n is the n^{th} term of the sequence, n is the term number, and C is a constant.

Substitute $n = 1$ and $a_1 = -9$ into the formula and solve for C.

$$-9 = 3(1) + C$$
$$C = -12$$

Therefore, the correct formula for the sequence is $\boldsymbol{a_n = 3n - 12}$.

7. Use $u(1) = 47$ and $u(n + 1) = u(n) + 4$, for $n = 1, 2, 3, \ldots$, to find a formula based on the position of each number.

$$u(1) = 47$$

$$\begin{aligned} u(2) &= u(1) + 4 \\ &= (47) + 4 \\ &= (47) + (4)(1) \end{aligned}$$

$$\begin{aligned} u(3) &= u(2) + 4 \\ &= (47 + 4) + 4 \\ &= (47) + (4)(2) \end{aligned}$$

$$\begin{aligned} u(4) &= u(3) + 4 \\ &= (47 + 4 + 4) + 4 \\ &= (47) + (4)(3) \\ &. \\ &. \\ &. \\ u(\text{n}) &= u(n - 1) + 4 \\ &= u(1) + 4(n - 1) \\ &= 47 + 4(n - 1) \\ &= 47 + 4n - 4 \\ &= 43 + 4n \end{aligned}$$

Apply this information to the first 4 terms.

$$u(1) = (43) + (4)(1) = 47$$
$$u(2) = (43) + (4)(2) = 51$$
$$u(3) = (43) + (4)(3) = 55$$
$$u(4) = (43) + (4)(4) = 59$$

Thus, the explicit equation is ***u*(*n*) = 43 + 4*n***.

8. The explicit formula for a geometric sequence is shown below.

$$t_n = t_1(r)^{n-1}$$

In the equation above, t_1 is the first term, *r* is the common ratio, and *n* = 1, 2, 3, ...

In this sequence, the first term is 3, and the common ratio is 5.

For instance, the value of t_3 can be found as shown below.

$$\begin{aligned} t_3 &= t_1(5)^2 \\ &= (3)(5)^2 \\ &= (3)(25) \\ &= 75 \end{aligned}$$

Therefore, the explicit formula is **t_n = 3(5)$^{n-1}$, where *n* = 1, 2, 3, ...**

9. The first term of this arithmetic sequence is:

$$8$$

Find the common difference in each of the numbers of the sequence. The common difference will always be the absolute value of the difference.

$$\begin{aligned} 32 - 24 &= 8 \\ 24 - 16 &= 8 \\ 16 - 8 &= 8 \end{aligned}$$

Therefore, each term is 8more than the term before it.

$$\begin{aligned} a_1 &= 8 \\ a_2 &= a_1 + 8 = 16 \\ a_3 &= a_2 + 8 = 24 \\ a_4 &= a_3 + 8 = 32 \end{aligned}$$

Now, write this as a recursive formula.

$$a_1 = 8 \text{ and } a_{n+1} = a_n + 8, \text{ where } n = 1, 2, 3, 4, \ldots$$

10. The explicit formula for a geometric sequence is *f*(*n*) = [*f*(1)](*r*)$^{n-1}$, where *r* is the common ratio, and *n* = 1, 2, 3,...

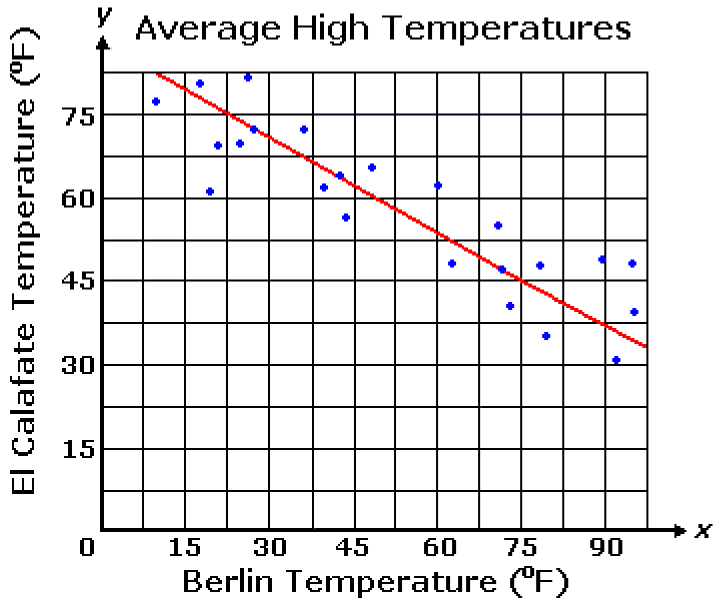
In this sequence, the common ratio is $\frac{1}{4}$.

For instance, $f(3) = [f(1)]\left(\frac{1}{4}\right)^2 = (4)\left(\frac{1}{4}\right)^2 = (4)\left(\frac{1}{16}\right) = \frac{1}{4}$.

Therefore, the explicit formula is $f(n) = 4\left(\frac{1}{4}\right)^{n-1}$, where *n* = 1, 2, 3,...

Algebra I: Scatter Plots

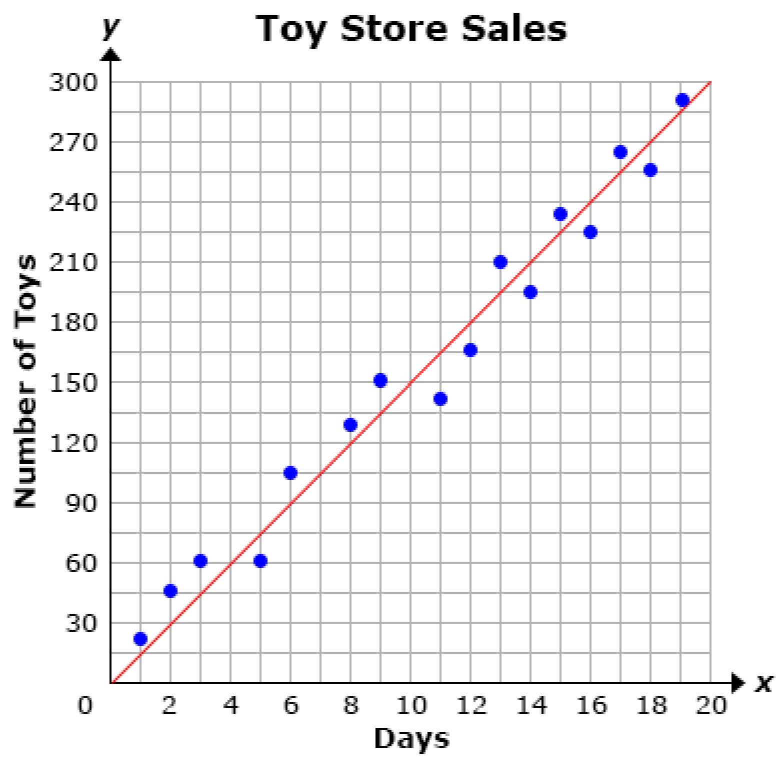
1. Berlin, Germany, and El Calafate, Argentina, are about the same distance from the equator. The graph below shows a line of best fit for data collected on the average high temperature in El Calafate as a function of the average high temperature in Berlin.



Which of the following is the equation of the line of best fit?

- **A.** $y = -\frac{7}{4}x + \frac{615}{7}$
- **B.** $y = -\frac{4}{7}x + 95$
- **C.** $y = -\frac{7}{4}x + 95$
- **D.** $y = -\frac{4}{7}x + \frac{615}{7}$

2. The graph below shows a line of best fit for data collected on the number of toys sold at a toy store since the opening of the store.



Based on the line of best fit, how many toys were sold 5 days after the store opened?

- **A.** 100
- **B.** 50
- **C.** 125
- **D.** 75

3. The profits of a high school baseball team are related to the number of fans that attend each game. A collection of data on the attendance (*x*) and profits (*y*) of 10 games produced the table below.

x	*y*
10	50
10	75
20	100
30	125
30	150
40	250
50	200
50	225
60	250
70	300

Using a residual plot, determine if the following model is a good fit for the data in the table above.

$$\hat{y} = 3.917x - 29.559$$

- **A.** No. The model is not a good fit because the residual plot has a random pattern.
- **B.** No. The model is not a good fit because the residual plot does not have a random pattern.
- **C.** There is not enough information to determine if the model is a good fit.
- **D.** Yes. The model is a good fit because the residual plot has a random pattern.

---

4. The table below shows the cost of renting a kayak at a local river for different amounts of time.

Number of Hours, *x*	1	2	3	4	5
Cost in Dollars, *C*	37	58	75	94	110

Which equation best models this set of data?

- **A.** $C = 65 - 2x^2$
- **B.** $C = x^2 - 4x + 42$
- **C.** $C = 20x + 19$
- **D.** $C = 18x + 21$

5. Rhonda started keeping track of how many words her daughter, Lily, knew once she was 12 months old. The number of words in her daughter's vocabulary every 3 months is shown in the table.

Lily's Vocabulary

Lily's Age (in months)	12	15	18	21	24
Number of Words	2	4	8	16	32

Which of the following functions would best model the data above?

- **A.** linear function
- **B.** quadratic function
- **C.** exponential function
- **D.** constant function

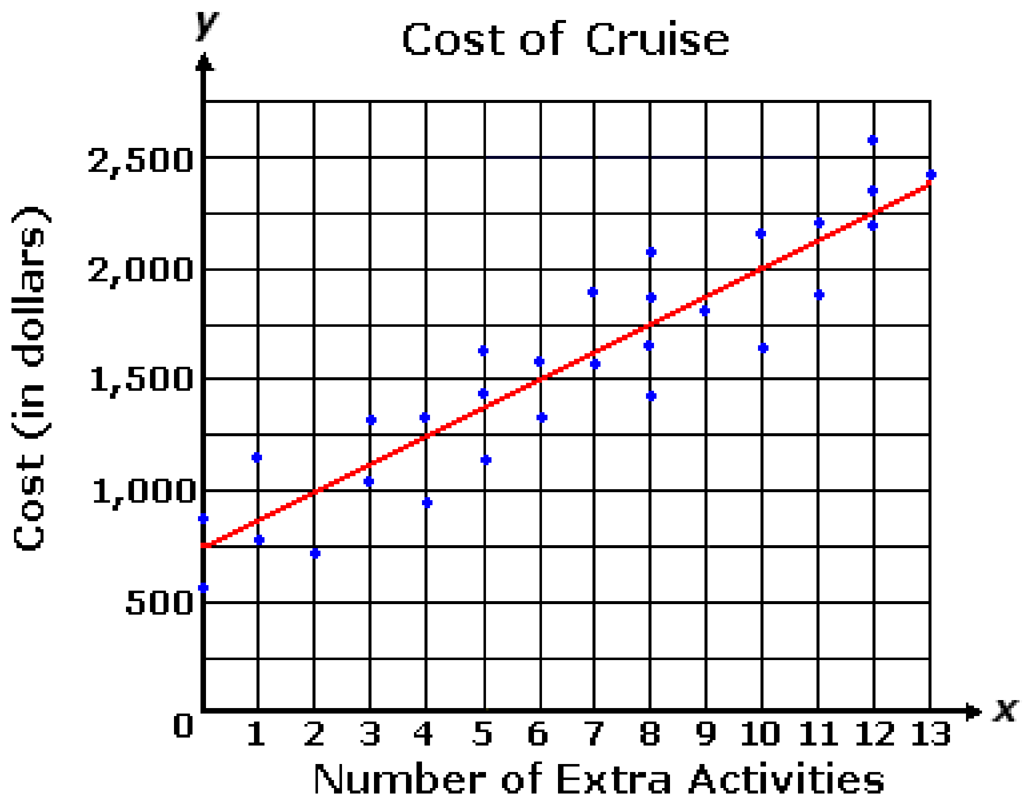
6. Ten people were chosen at random and surveyed. The table below shows the number hours per week that the participants exercise (X) and the number of pounds that they are overweight (Y).

Time Spent Working Out per Week *(in hours)*	**lbs Overweight**
X	**Y**
0.5	20
1	28
1.5	8
2	15
2.5	8
3	5
3.5	2
4	2
4.5	0
5	1

Using technology, compute the correlation coefficient, r, and determine if there is a correlation between the variables.

- **A.** $r \approx 0.53$; strong correlation
- **B.** $r \approx -0.53$; weak correlation
- **C.** $r \approx 0.87$; weak correlation
- **D.** $r \approx -0.87$; strong correlation

7. The graph below shows a line of best fit on data collected on the cost of a cruise in relation to the number of extra activities added to the cruise.



<--JJ Jan2016--!>

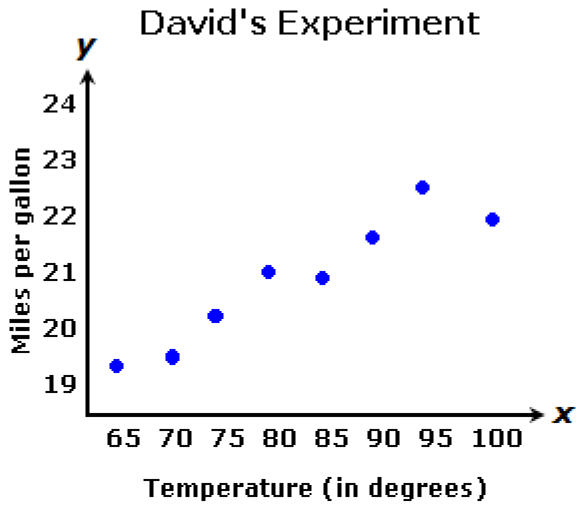
The equation of the line of best fit is as shown below.

$$y = 125x + 750$$

What does the *y*-intercept of this line represent?

- ◯ **A.** the cost of the cruise before adding any activities
- ◯ **B.** the cost per activity
- ◯ **C.** the total cost of all activities
- ◯ **D.** the number of activities added

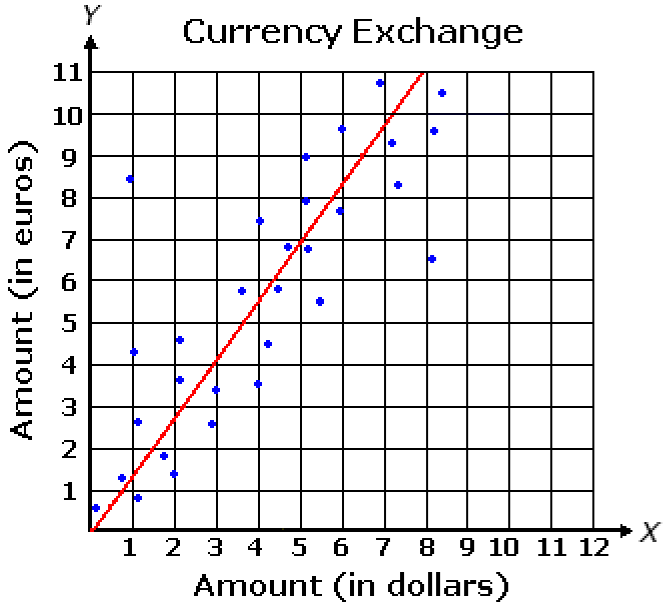
8. David wondered if the temperature outside affects his gas mileage. He recorded the temperature and his gas mileage over the last few months. His data is shown below.



Which of the following is true?

- **A.** There is a negative correlation between temperature and gas mileage.
- **B.** There is a positive correlation between temperature and gas mileage.
- **C.** There is no correlation between temperature and gas mileage.
- **D.** There is both a positive correlation and a negative correlation between temperature and gas mileage.

9. The graph below shows a line of best fit for the number of euros customers received from exchanging U.S. dollars. The transaction includes a conversion fee.



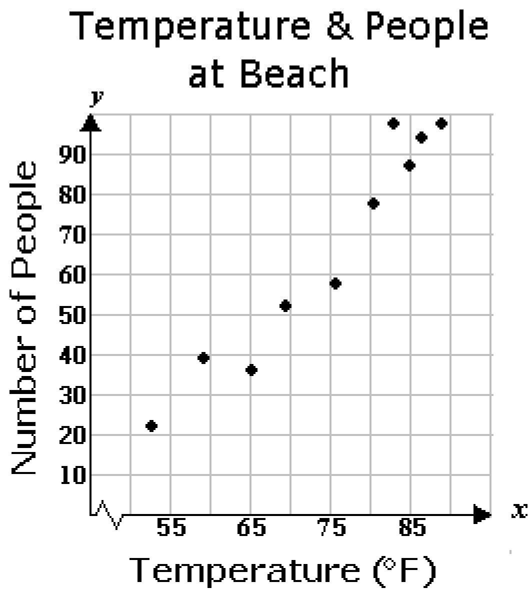
The equation for the line of best fit is shown below.

$$y = \frac{7}{5}x$$

What does the slope of this line represent?

- **A.** the conversion fee
- **B.** the number of U.S. dollars converted to euros
- **C.** the number of U.S. dollars equal to one euro
- **D.** the number of euros equal to one U.S. dollar

10. Tony, the lifeguard at Waununa Beach, recorded the temperature and the number of people on his stretch of beach over the course of several weeks. He collected data at noon on Tuesdays and Saturdays during the five weeks between Mother's Day and Father's Day, as shown in the graph below.



Which of the following is a valid conclusion?

- **A.** The increase in temperature caused an increase in the number of people at the beach.
- **B.** There is no correlation between the temperature and the number of people at the beach.
- **C.** The increase in the number of people at the beach is correlated to, but not caused by, the increase in temperature.
- **D.** No conclusion can be drawn about the correlation or causation of the temperature and the number of people at the beach.

Answers: Scatter Plots

1. D
2. D
3. B
4. D
5. C
6. D
7. A
8. B
9. D
10. C

Explanations

1. The equation of a line can be written as $y = mx + b$, where m is the slope and b is the y-intercept.

The points (22.5,75) and (75,45) both lie on the line of best fit. Use these points to find the slope of the line.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{45 - 75}{75 - 22.5} \\ &= -\frac{4}{7} \end{aligned}$$

To find the equation of the line of best fit, substitute the slope and the point (75,45) into the point-slope equation, and solve for y.

$$\begin{aligned} y - 45 &= -\frac{4}{7}(x - 75) \\ y - 45 &= -\frac{4}{7}x + \frac{300}{7} \\ y &= -\frac{4}{7}x + \frac{615}{7} \end{aligned}$$

2. In this case, the number of days since the store opened is on the x-axis, and the number of toys sold is on the y-axis.

Use the line of best fit to find the value of y when x equals 5 days.

When x equals 5 days, y equals **75** toys.

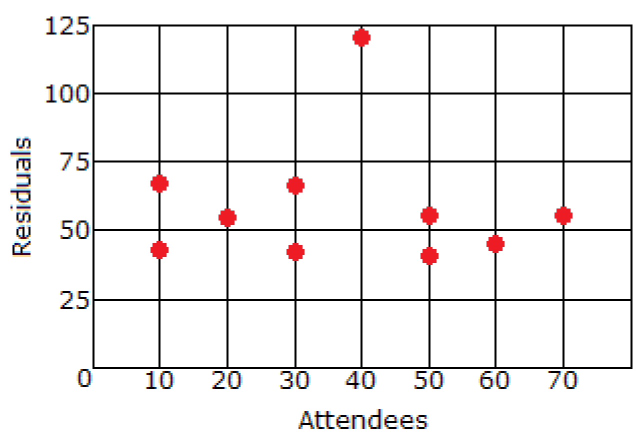
3. Residuals, e, are the differences in actual data values and the corresponding predicted data values produced by a regression model.

$$e = y - \hat{y}$$

In order to create a residual plot, create a table to calculate the predicted *y*-values and the values of the residuals.

x	y	$\hat{y} = 3.917x - 29.559$	$e = y - \hat{y}$
10	50	$3.917(10) - 29.559 = 9.611$	$50 - 9.611 = 40.389$
10	75	$3.917(10) - 29.559 = 9.611$	$75 - 9.611 = 65.389$
20	100	$3.917(20) - 29.559 = 48.781$	$100 - 48.781 = 51.219$
30	125	$3.917(30) - 29.559 = 87.951$	$125 - 87.951 = 37.049$
30	150	$3.917(30) - 29.559 = 87.951$	$150 - 87.951 = 62.049$
40	250	$3.917(40) - 29.559 = 127.12$	$250 - 127.12 = 122.88$
50	200	$3.917(50) - 29.559 = 166.29$	$200 - 166.29 = 33.71$
50	225	$3.917(50) - 29.559 = 166.29$	$225 - 166.29 = 58.71$
60	250	$3.917(60) - 29.559 = 205.46$	$250 - 205.46 = 44.54$
70	300	$3.917(70) - 29.559 = 244.63$	$300 - 244.63 = 55.37$

Create a residual plot using the residuals calculated in the table above.



The residual plot has a linear pattern with an outlier; therefore, the model is not a good fit.

4. The first step in a problem like this is to determine what type of equation would model the data. To do this, look at the differences between the numbers in the second row.

58 - 37 = 21

75 - 58 = 17

94 - 75 = 19

110 - 94 = 16

Since all of the differences are approximately equal, the numbers can be modeled by a linear equation. This leaves two possible answer choices.

Now that there are only two possible answer choices, plug in numbers from the table to see which of the linear equations works best.

$C = 20x + 19$	$C = 18x + 21$
$C = 20(1) + 19 = 39$	$C = 18(1) + 21 = 39$
$C = 20(2) + 19 = 59$	$C = 18(2) + 21 = 57$
$C = 20(3) + 19 = 79$	$C = 18(3) + 21 = 75$
$C = 20(4) + 19 = 99$	$C = 18(4) + 21 = 93$
$C = 20(5) + 19 = 119$	$C = 18(5) + 21 = 111$

The values from $C = 18x + 21$ are closest to those in the table.

Therefore, the equation $\boldsymbol{C = 18x + 21}$ best models the data.

5.

To determine the type of function needed, look at the differences between the values in the second row (the first differences) and also look at the differences between the first differences (the second differences).

First Difference	**Second Difference**
4 - 2 = 2	4 - 2 = 2
8 - 4 = 4	8 - 4 = 4
16 - 8 = 8	16 - 8 = 8
32 - 16 = 16	

The first difference values are not constant, so the equation that best models the data will not be linear. The second difference values are not constant, so the equation that best models the data will not be quadratic.

Determine if the terms have a common ratio by dividing each term by the previous term.

$$4 \div 2 = 2$$
$$8 \div 4 = 2$$
$$16 \div 8 = 2$$
$$32 \div 16 = 2$$

Since the terms have a common ratio, an **exponential function** best models this set of data.

6. First, calculate the correlation coefficient using technology.

$$r = -0.86589\ldots \approx -0.87$$

Since the coefficient is close to -1, there is a strong correlation between the two variables.

Therefore, the correct answer is shown below.

r ≈-0.87 ; strong correlation

7. The *y*-intercept is the value of the quantity on the *y*-axis, when the quantity on the *x*-axis is zero.

In this case, the quantity on the *x*-axis is the number of activities added to the cruise. When this is zero, there are no activities added to the cruise.

Therefore, the *y*-intercept is **the cost of the cruise before adding any activities**.

8. The temperature is shown on the *x*-axis, and the gas mileage is shown on the *y*axis. As the temperature outside increases, the higher the gas mileage on his car. (High gas mileage is preferred to low gas mileage.) The dots on the scatter plot go up and to the right.

Therefore, **there is a positive correlation between temperature and gas mileage**.

9. The slope of a line is the change in the quantity on the *y*-axis per unit of the quantity on the *x*-axis.

In this case, the quantity on the *y*-axis is the amount in euros, and the quantity on the *x*-axis is the amount in U.S. dollars.

Therefore, the slope represents the number of euros per one U.S. dollar, or **the number of euros equal to one U.S. dollar**.

10. Causation between variables indicates that one of the variables causes the other variable.

The temperature might be one reason for an increased number of people, but, based on the information given, it cannot be determined to be the sole cause. Other factors might contribute, like the fact that school is over, more people are vacationing, etc.

However, Tony could conclude that more people went to the beach as the temperature increased.

Therefore, **the increase in the number of people at the beach is correlated to, but not caused by, the increase in temperature**.

Algebra I: Simplify Expressions

1. Factor the following expression completely.

$$256x^{10} - x^6$$

- **A.** $x^6(4x - 1)^2(16x^2 + 1)$
- **B.** $x^6(256x^4 - 1)$
- **C.** $x^6(16x^2 - 1)(16x^2 + 1)$
- **D.** $x^6(4x - 1)(4x + 1)(16x^2 + 1)$

---

2. Simplify the following expression.

$$\frac{3x^2 - 31x + 10}{x - 3x^2}$$

- **A.** $-\frac{x - 10}{x}$
- **B.** $\frac{x + 10}{x}$
- **C.** $\frac{x - 10}{3x - 1}$
- **D.** $-\frac{3x - 1}{x}$

---

3. Factor the following expression completely.

$$x^4 - 81$$

- **A.** $(x^2 - 9)(x^2 + 9)$
- **B.** $(x - 3)(x + 3)(x^2 + 9)$
- **C.** $(x - 3)(x^3 + 27)$
- **D.** $(x - 3)(x + 3)(x - 3)(x + 3)$

---

4. Factor the following expression completely.

$$4x^2 - 49y^2$$

- **A.** $(4x + 49y)(4x - 49y)$
- **B.** $(2x + 7y)(2x - 7y)$
- **C.** $(4x - 49y)^2$
- **D.** $(2x - 7y)^2$

5. Factor the following expression completely.

$$x^4 - y^4$$

- **A.** $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$
- **B.** $(x - y)^2(x + y)^2$
- **C.** $(x - y)^4$
- **D.** $(x - y)(x + y)\left(x^2 + y^2\right)$

---

6. Simplify the following expression.

$$8z(4y + 9z) + 72z$$

- **A.** $144z^2 + 32yz$
- **B.** $144z^3 + 32yz$
- **C.** $72z^2 + 32yz - 72z$
- **D.** $72z^2 + 32yz + 72z$

---

7. Factor the following expression completely.

$$36x^{14} - x^{12}$$

- **A.** $x^{12}(6x - 1)(6x + 1)$
- **B.** $\left(6x^7 - x^6\right)^2$
- **C.** $x^{12}(6x - 1)^2$
- **D.** $x^{12}\left(36x^2 - 1\right)$

---

8. Simplify the following expression:

$$\left(\sqrt{11x} + \sqrt{5x}\right)^2$$

- **A.** $16x + 2x\sqrt{55}$
- **B.** $16x^2 + 2x\sqrt{55}$
- **C.** $16x + 110\sqrt{x}$
- **D.** $16x$

9. Simplify.

$$\frac{10x(8y^3)}{x^2y^4} - \frac{7x^2 - 5x^2}{x^3y}$$

- A. $80xy - 12x^2$
- B. $\frac{78}{xy}$
- C. $80xy - 2x^2$
- D. $\frac{6}{xy}$

---

10. Simplify the following expression.

$$\frac{(8x + 5y)^2}{y^2} + 7$$

- A. $64x^2y^2 + 80xy + 32$
- B. $\frac{64x^2}{y^2} + 32$
- C. $\frac{64x^2}{y^2} + \frac{80x}{y} + 32$
- D. $\frac{8x^2}{y^2} + \frac{80x}{y} + 12$

Answers: Simplify Expressions

1. D
2. A
3. B
4. B
5. D
6. D
7. A
8. A
9. B
10. C

Explanations

1. First, factor out x^6, and then factor using the difference of two squares.

$$\begin{aligned} 256x^{10} - x^6 &= x^6(256x^4 - 1) \\ &= x^6\left((16x^2)^2 - (1)^2\right) \\ &= x^6(16x^2 - 1)(16x^2 + 1) \\ &= x^6\left((4x)^2 - (1)^2\right)(16x^2 + 1) \\ &= x^6(4x - 1)(4x + 1)(16x^2 + 1) \end{aligned}$$

2. First, factor the polynomials in the numerator and the denominator.

$$\frac{3x^2 - 31x + 10}{x - 3x^2} = \frac{(x - 10)(3x - 1)}{-x(3x - 1)}$$

Then, eliminate the common factors.

$$\frac{(x - 10)\cancel{(3x - 1)}}{-x\cancel{(3x - 1)}} = -\frac{x - 10}{x}$$

3. Factor using the difference of two squares.

$$\begin{aligned} x^4 - 81 &= (x^2)^2 - (9)^2 \\ &= (x^2 - 9)(x^2 + 9) \\ &= \left((x)^2 - (3)^2\right)(x^2 + 9) \\ &= (x - 3)(x + 3)(x^2 + 9) \end{aligned}$$

4. Factor using the difference of two squares.

$$4x^2 - 49y^2 = (2x)^2 - (7y)^2$$
$$= \mathbf{(2x + 7y)(2x - 7y)}$$

5. Factor using the difference of two squares.

$$x^4 - y^4 = \left(x^2\right)^2 - \left(y^2\right)^2$$
$$= \left(x^2 - y^2\right)\left(x^2 + y^2\right)$$
$$= \left((x)^2 - (y)^2\right)\left(x^2 + y^2\right)$$
$$= (x - y)(x + y)\left(x^2 + y^2\right)$$

6.

$$8z\left(4y + 9z\right) + 72z = 32yz + 72z^2 + 72z$$
$$= 72z^2 + 32yz + 72z$$

7. First, factor out x^{12}, and then factor using the difference of two squares.

$$36x^{14} - x^{12} = x^{12}\left(36x^2 - 1\right)$$
$$= x^{12}\left((6x)^2 - (1)^2\right)$$
$$= x^{12}(6x - 1)(6x + 1)$$

8.

To simplify the expression, begin by rewriting the squared binomial without an exponent.

$$\left(\sqrt{11x} + \sqrt{5x}\right)\left(\sqrt{11x} + \sqrt{5x}\right)$$

Multiply the binomials and simplify by combining like terms.

$$\left(\sqrt{11x} + \sqrt{5x}\right)\left(\sqrt{11x} + \sqrt{5x}\right) = 11x + \sqrt{55x^2} + \sqrt{55x^2} + 5x$$
$$= 11x + x\sqrt{55} + x\sqrt{55} + 5x$$
$$= 16x + 2x\sqrt{55}$$

9. Simplify the first part of the expression.

$$\frac{10x(8y^3)}{x^2y^4} = \frac{80xy^3}{x^2y^4} = \frac{80}{xy}$$

Simplify the second part of the expression.

$$\frac{7x^2 - 5x^2}{x^3y} = \frac{2x^2}{x^3y} = \frac{2}{xy}$$

Combine like terms.

$$\frac{80}{xy} - \frac{2}{xy} = \frac{78}{xy}$$

10.

$$\begin{aligned}\frac{(8x + 5y)^2}{y^2} + 7 &= \frac{(8x + 5y)(8x + 5y)}{y^2} + 7\\ &= \frac{64x^2 + 80xy + 25y^2}{y^2} + 7\\ &= \frac{64x^2}{y^2} + \frac{80xy}{y^2} + \frac{25y^2}{y^2} + 7\\ &= \frac{64x^2}{y^2} + \frac{80x}{y} + 25 + 7\\ &= \frac{64x^2}{y^2} + \frac{80x}{y} + 32\end{aligned}$$

Algebra I: Quadratic Expressions

1. Factor the following polynomial completely.

$$-0.7x^2 - 4.9x + 5.6$$

- **A.** $0.7(x + 8)(x - 1)$
- **B.** $-0.7(x + 8)(x - 1)$
- **C.** $-0.7(x^2 + 7x - 8)$
- **D.** $-0.7(x + 8)(x + 1)$

---

2. Factor the polynomial below.

$$x^2 + 6x + 8$$

- **A.** $(x + 2)(x + 4)$
- **B.** $(x + 2)(x - 4)$
- **C.** $(x - 2)(x - 4)$
- **D.** $(x - 2)(x + 4)$

---

3. Factor the following expression completely.

$$3x^2 - 108$$

- **A.** $(3x + 6)(x - 6)$
- **B.** $3(x + 6)(x - 6)$
- **C.** $3(x^2 - 36)$
- **D.** $3(x - 6)^2$

---

4. What is the factored form of the following expression?

$$x^2 - 32x + 256$$

- **A.** $(x + 16)^2$
- **B.** $(x + 16)(x - 16)$
- **C.** $(x - 16)^2$
- **D.** $16(x - 16)^2$

5. Which expression is the factored equivalent of $x^2 - 36$?

- **A.** $(x + 4)(x - 9)$
- **B.** $(x - 6)(x + 6)$
- **C.** $(x + 3)(x - 12)$
- **D.** $(x - 6)^2$

---

6. What is the factored form of the following expression?

$$25x^2 + 20x + 4$$

- **A.** $25(x + 2)^2$
- **B.** $(5x + 2)(2x - 5)$
- **C.** $(5x + 2)^2$
- **D.** $(5x + 2)(5x - 2)$

---

7. Determine which answer choice below is equivalent to the given expression by completing the square.

$$-x^2 - 6x + 4$$

- **A.** $-(x + 3)^2 - 5$
- **B.** $-(x + 3)^2 + 4$
- **C.** $-(x + 3)^2 - 9$
- **D.** $-(x + 3)^2 + 13$

---

8. Factor the polynomial below.

$$x^2 - 6x + 9$$

- **A.** $(x - 6)^2$
- **B.** $2x^2 - 9$
- **C.** $x^2 - 9$
- **D.** $(x - 3)^2$

9. Which expression is the factored equivalent of $x^2 - 5x + 4$?

- **A.** $(x + 4)(x + 1)$
- **B.** $(x - 1)(x - 4)$
- **C.** $(x + 1)(x - 4)$
- **D.** $(x - 3)(x - 1)$

---

10. Factor the following polynomial.

$$12x^2 + 24x$$

- **A.** $-12x(x - 2)$
- **B.** $12x(x + 2)$
- **C.** $12x(x - 2)$
- **D.** $12x(x + 24)$

Answers: Quadratic Expressions

1. B
2. A
3. B
4. C
5. B
6. C
7. D
8. D
9. B
10. B

Explanations

1. First, factor out the greatest common factor of the coefficients. The greatest common factor of -0.7, -4.9, and 5.6 is -0.7.

$$-0.7x^2 - 4.9x + 5.6 = -0.7(x^2 + 7x - 8)$$

Next, factor the resulting trinomial.

$$-0.7(x^2 + 7x - 8) = \mathbf{-0.7(\boldsymbol{x} + 8)(\boldsymbol{x} - 1)}$$

2. Any polynomial in the form $x^2 + bx + c$, where b and c are positive, can be factored using the formula below.

$$x^2 + (m + n)x + mn = (x + m)(x + n)$$

In this case, the sum of m and n is equal to b, or 6, and the product of m and n is equal to c, or 8. Find values of m and n that satisfy these conditions.

$$2 + 4 = 6$$
$$2 \cdot 4 = 8$$

Substitute m = 2 and n = 4 into the formula to factor the polynomial.

$$x^2 + 6x + 8 = \mathbf{(\boldsymbol{x} + 2)(\boldsymbol{x} + 4)}$$

3. First, factor out a 3, and then factor using the difference of two squares.

$$\begin{aligned} 3x^2 - 108 &= 3(x^2 - 36) \\ &= \mathbf{3(\boldsymbol{x} + 6)(\boldsymbol{x} - 6)} \end{aligned}$$

4. The expression is in the form $a^2x^2 - 2abx + b^2$, where $a = 1$ and $b = 16$. Therefore, it can be factored as a perfect square trinomial.

$$a^2x^2 - 2abx + b^2 = (ax - b)^2$$
$$x^2 - 32x + 256 = \mathbf{(x - 16)^2}$$

5. The difference of two squares can be factored according to the equation below.

$$x^2 - a^2 = (x - a)(x + a)$$

In the polynomial $x^2 - 36$, $a = 6$. Use this information to factor the polynomial.

$$x^2 - 36 = \mathbf{(x - 6)(x + 6)}$$

6. The expression is in the form $a^2x^2 + 2abx + b^2$, where $a = 5$ and $b = 2$. Therefore, it can be factored as a perfect square trinomial.

$$a^2x^2 + 2abx + b^2 = (ax + b)^2$$
$$25x^2 + 20x + 4 = \mathbf{(5x + 2)^2}$$

7. Complete the square for the given expression.

$$\begin{aligned} -x^2 - 6x + 4 &= -\left(x^2 + 6x\right) + 4 \\ &= -\left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) + 4 + \left(\frac{6}{2}\right)^2 \\ &= -\left(x^2 + 6x + 3^2\right) + 4 + (3)^2 \\ &= -\left(x^2 + 6x + 9\right) + 4 + 9 \\ &= -(x + 3)^2 + 13 \end{aligned}$$

8. The polynomial $ax^2 + bx + c$ is a perfect square trinomial if $(b \div 2)^2$ is equal to c.

$$(-6 \div 2)^2 = 9$$

Therefore, the polynomial is a perfect square trinomial.

The formula for the square of the binomial $(x - y)$ is shown below.

$$(x - y)^2 = x^2 - 2xy + y^2$$

In this case, $y = 3$. Use this formula to factor the polynomial.

$$x^2 - 6x + 9 = \mathbf{(x - 3)^2}$$

9. The polynomial $x^2 - 5x + 4$ is in the form $ax^2 + bx + c$, where $a = 1$, $b = -5$, and $c = 4$. Find the factors of a and c.

1: 1

4: 1, 2, 4

Form two pairs of factors from one factor of *a* and one factor of *c*, so that when the factors are multiplied and then added together (because *c* is positive), they equal the absolute value of *b*.

$$(1 \cdot 4) + (1 \cdot 1) = 5$$

Use these numbers to form the factors of the polynomial. Because *b* is negative, both factors will have subtraction signs.

$$(x - 1)(x - 4)$$

Therefore, $\mathbf{(x - 1)(x - 4)}$ is the factored equivalent of $x^2 - 5x + 4$.

10. First, find the greatest common factor of the two coefficients.

The greatest common factor of 12 and 24 is 12. Factor this out of the polynomial.

$$12x^2 + 24x = 12(x^2 + 2x)$$

Next, find the greatest common factor of the variables.

The greatest common factor of x^2 and x is x. Factor this out of the polynomial.

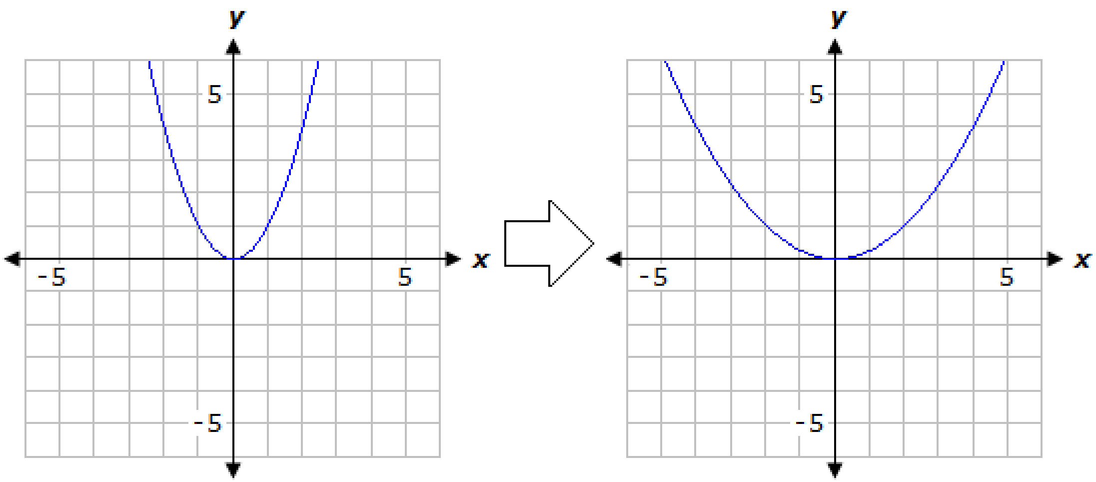
$$12(x^2 + 2x) = \mathbf{12x(x + 2)}$$

Algebra I: Function Transformations

1. What happens to the graph of $y = |x|$ when the equation changes to $y = |x| - 7$?

- **A.** The graph shifts left 7 units.
- **B.** The graph shifts up 7 units.
- **C.** The graph shifts right 7 units.
- **D.** The graph shifts down 7 units.

2.



$f(x) = x^2$ $g(x) = ?$

Which of the following is equal to $g(x)$?

- **A.** $\frac{1}{4}x^2$
- **B.** $(x+1)^2$
- **C.** $\left(\frac{1}{4}x\right)^2$
- **D.** $\frac{1}{2}x^2$

3. Each statement describes a transformation of the graph of $y = x^2$. Which statement correctly describes the graph of the equation shown below?

$$y = 3(x + 7)^2 + 10$$

- **A.** It is the graph of $y = x^2$ vertically compressed, and then translated 7 units up and 10 units to the right.
- **B.** It is the graph of $y = x^2$ vertically stretched, and then translated 7 units down and 10 units to the right.
- **C.** It is the graph of $y = x^2$ vertically stretched, and then translated 10 units up and 7 units to the left.
- **D.** It is the graph of $y = x^2$ vertically compressed, and then translated 10 units down and 7 units to the left.

---

4. If the graph of $f(x) = |x|$ is shifted down 9 units, what would be the equation of the new graph?

- **A.** $g(x) = |x| - 9$
- **B.** $g(x) = |x - 9|$
- **C.** $g(x) = |x + 9|$
- **D.** $g(x) = |x| + 9$

---

5. Which of the following functions will produce a graph that is a vertically compressed version of the graph of $y = x^2$?

- **A.** $y = 2x^2$
- **B.** $y = x^{-2}$
- **C.** $y = -x^2$
- **D.** $y = \frac{1}{2}x^2$

6. Each statement describes a transformation of the graph of $y = |x|$. Which statement correctly describes the graph of $y = |x + 5| - 2$?

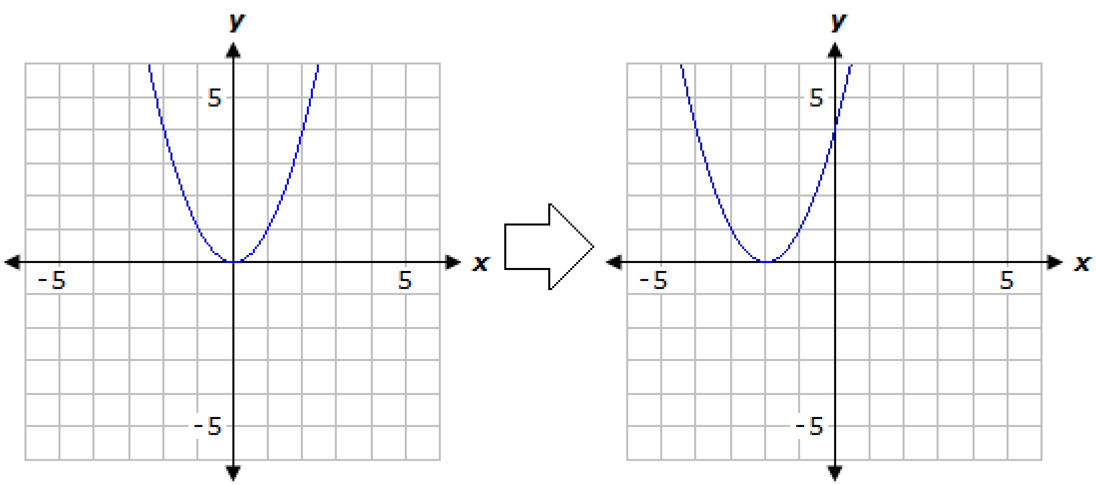
- **A.** It is the graph of $y = |x|$ translated 2 units down and 5 units to the right.
- **B.** It is the graph of $y = |x|$ translated 5 units up and 2 units to the left.
- **C.** It is the graph of $y = |x|$ translated 5 units up and 2 units to the right.
- **D.** It is the graph of $y = |x|$ translated 2 units down and 5 units to the left.

---

7. Each statement describes a transformation of the graph of $y = x^2$. Which statement correctly describes the graph of $y = (x - 4)^2 + 5$?

- **A.** It is the graph of $y = x^2$ translated 4 units down and 5 units to the right.
- **B.** It is the graph of $y = x^2$ translated 4 units up and 5 units to the right.
- **C.** It is the graph of $y = x^2$ translated 5 units up and 4 units to the right.
- **D.** It is the graph of $y = x^2$ translated 5 units up and 4 units to the left.

8.



$f(x) = x^2$ $g(x) = ?$

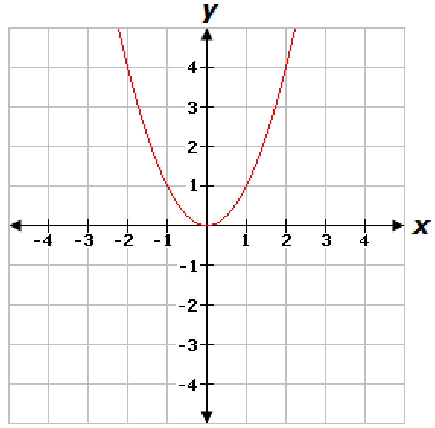
Which of the following is equal to $g(x)$?

- **A.** $(x + 2)^2$
- **B.** $x^2 - 2$
- **C.** $(x - 2)^2$
- **D.** $x^2 + 2$

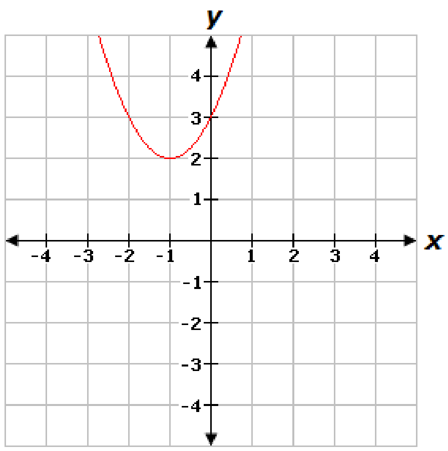
9. In which direction must the graph of $f(x) = x^2$ be shifted to produce the graph of $g(x) = x^2 + 9$?

- **A.** up
- **B.** down
- **C.** right
- **D.** left

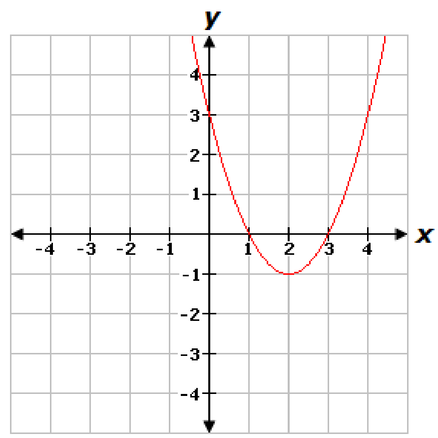
10.



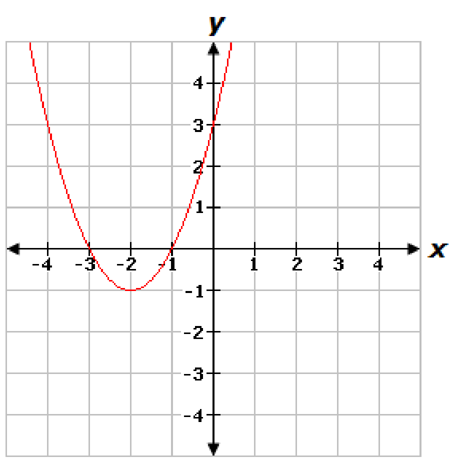
The function $f(x) = x^2$ is graphed above. Which of the graphs represents the function $g(x) = (x + 2)^2 - 1$?



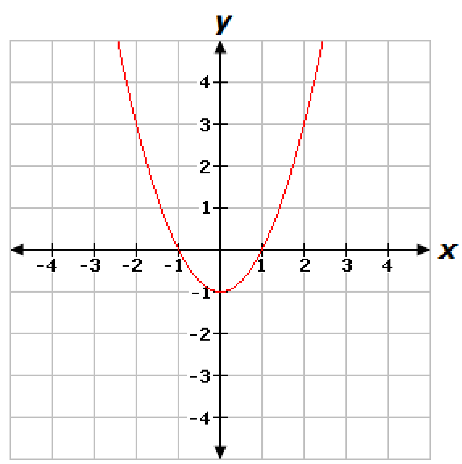
W.



X.



Y.



Z.

- ◯ **A.** X
- ◯ **B.** Z
- ◯ **C.** W
- ◯ **D.** Y

Answers: Function Transformations

1. D
2. A
3. C
4. A
5. D
6. D
7. C
8. A
9. A
10. D

Explanations

1. When a number is added or subtracted outside of the absolute value sign, it causes a vertical shift to the absolute value function. If the number is added, the graph shifts up, and if the number is subtracted, the graph shifts down.

Since 7 is subtracted from the equation, **the graph shifts down 7 units**.

2. Consider the graph of a function $r(x)$ and the real number A.

$A \cdot r(x)$ with $A > 1$ stretches the graph vertically by a factor of A.

$A \cdot r(x)$ with $0 < A < 1$ compresses the graph vertically by a factor of A.

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3. Consider the function $y = f(x)$ with the real numbers $a > 0$, $h > 0$, and $r > 0$.

$y = f(x) + r$	shifts the graph of $f(x)$ up r units
$y = f(x) - r$	shifts the graph of $f(x)$ down r units
$y = f(x + h)$	shifts the graph of $f(x)$ to the left h units
$y = f(x - h)$	shifts the graph of $f(x)$ to the right h units

$y = a \cdot f(x)$, where $a > 1$	stretches the graph of $f(x)$ vertically by a factor of a
$y = a \cdot f(x)$, where $0 < a < 1$	compresses the graph of $f(x)$ vertically by a factor of a

The given equation can be seen as $y = f(x) = x^2$ undergoing three transformations.

$$y = 3f(x + 7) + 10 = 3(x + 7)^2 + 10$$

Therefore, the given equation represents **the graph of $y = x^2$ vertically stretched, and then translated 10 units up and 7 units to the left**.

4. A vertical shift means a change will occur outside the function. To find the equation that would represent the graph of $f(x)$ shifted down 9 units, find $f(x) - 9$.

$$g(x) = f(x) - 9 = \mathbf{|x| - 9}$$

5. Given a function $y = f(x)$, the equation of the function when vertically stretched or compressed will be $y = a \cdot f(x)$, where $a > 0$.

If $a > 1$, the original graph will have been vertically stretched.

If $a < 1$, the original graph will have been vertically compressed.

Therefore, if the graph of $y = x^2$ is vertically compressed, the function will become $y = a \cdot x^2$, where $a < 1$.

The only answer choice that fits this description is given below.

$$y = \frac{1}{2}x^2$$

6. Consider the function $y = f(x)$ with the real numbers $h > 0$ and $r > 0$.

$y = f(x) + r$	shifts the graph of $f(x)$ up r units
$y = f(x) - r$	shifts the graph of $f(x)$ down r units
$y = f(x + h)$	shifts the graph of $f(x)$ left h units
$y = f(x - h)$	shifts the graph of $f(x)$ right h units

The given equation $y = |x + 5| - 2$ can be seen as $y = f(x) = |x|$ undergoing two transformations.

$$y = f(x + 5) - 2 = |x + 5| - 2$$

Therefore, the given equation represents **the graph of $y = |x|$ translated 2 units down and 5 units to the left**.

7. Consider the function $y = f(x)$ with the real numbers $h > 0$ and $r > 0$.

$y = f(x) + r$	shifts the graph of $f(x)$ up r units
$y = f(x) - r$	shifts the graph of $f(x)$ down r units
$y = f(x + h)$	shifts the graph of $f(x)$ left h units
$y = f(x - h)$	shifts the graph of $f(x)$ right h units

The given equation $y = (x - 4)^2 + 5$ can be seen as $y = f(x) = x^2$ undergoing two transformations.

$$y = f(x - 4) + 5 = (x - 4)^2 + 5$$

Therefore, the given equation represents **the graph of $y = x^2$ translated 5 units up and 4 units to the right**.

8. Consider the graph of a function $r(x)$ with real numbers k and h.

$r(x) + k$	shifts the graph up k units
$r(x) - k$	shifts the graph down k units
$r(x + h)$	shifts the graph to the left h units
$r(x - h)$	shifts the graph to the right h units

The graph of $g(x)$ is the graph of $f(x)$ shifted to the left 2 units.

Therefore, $g(x) = f(x + 2) = \mathbf{(x + 2)^2}$.

9. Notice that $g(x) = f(x) + 9$. Since the change happens outside the function, the shift must be a vertical shift. Since the number is being added to x^2, the graph of $g(x)$ was created by shifting the graph of $f(x)$ 9 units **up**.

10. Consider the graph of a function $r(x)$ with real numbers k and h.

$r(x) + k$	shifts the graph k units up
$r(x) - k$	shifts the graph k units down
$r(x + h)$	shifts the graph h units to the left
$r(x - h)$	shifts the graph h units to the right

The graph of $g(x) = (x + 2)^2 - 1$ is the graph of $f(x) = x^2$ shifted 1 unit down and 2 units to the left. Therefore, the correct graph is **Y**.